Composite Dark Matter with Invisible Light from Almost-Commutative Geometry

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Almost commutative geometry offers a specific way to unify general relativity, quantum mechanics and gauge symmetries. The AC-model of elementary particles, arising on this way, naturally embeds the Standard model and predicts doubly charged AC-leptons, anion-like A^{--} and cathion-like C^{++}, which can bind in WIMP-like (AC)-atoms, being a nontrivial candidate for cosmological dark matter. This state is reached in the early Universe along a tail of more manifest secondary frozen blocks. They should be now here polluting the surrounding matter. The main secondary relics are C^{++} "anomalous helium" and a bound system of A^{--} with an ordinary helium ion $(^4He)^{++}$, which is able to attract and capture (in the *first three minutes*) all the free A^{--} fixing them into a neutral *OLe-helium* (OHe) nuclear interacting "atom" $(^4He^{++}A^{--})$. The model naturally involves a new $U(1)$ gauge interaction, possessed only by the AC-leptons and providing a Coulomb-like attraction between them. This attraction stimulates the effective $A - C$ recombination into AC-atoms inside dense matter bodies (stars and planets), resulting in a decrease of anomalous isotopes below the experimental upper limits. *OLe-helium* pollution of terrestrial matter and (OHe) catalysis of nuclear reactions in it is one of the exciting problems (or advantages?) of the present model.

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The problem of the of new leptons, being among the most important in modern high energy physics, has acquired recently an interesting cosmological aspect. New heavy leptons may be sufficiently long-living to represent a new stable form of matter and even to offer a nontrivial solution for cosmological dark matter problem. At the present there are at least three main elementary particle frames for Heavy Stable Quarks and Leptons and their cosmological impact: (a) A fourth generation with heavy a stable U-Quark and a neutral Lepton (neutrino) (above half the Z-Boson mass)[1], [2], [3], [4], [5], [6] ; see also [7, 8, 9], [10], offering a possibility of an atom-like bound state $[^4He^{++}(\bar{U}\bar{U}\bar{U})^{--}]$ as a specific nuclear-interacting candidate for dark matter [11]; (b) A Glashow's "Sinister" tera-U-quark and a tera-electron, whose atomic bound states may be the dominant dark matter [12, 13] and (c) the possibility of doubly charged AC-leptons[49] A^{--} and C^{++} recently revealed in [14] for the AC-model [15], following from the approach of almost-commutative geometry by Alain Connes [16].

This latter option of the AC-model provides dark matter in the form of evanescent bound AC-leptonic "atoms" (AC) and can avoid most of troubles of atom-like composite dark matter scenarios, if the AC-leptons possess an additional new $U(1)$ gauge interaction [14]. We shall address here our attention to the physical grounds of the AC-model, on the conditions under which it gives rise to new $U(1)$ interaction for the AC-leptons and on the self-consistency of the cosmological dark matter scenario, based on this approach.

The AC-model [15] appeared as a realistic elementary particle model, based on the specific approach of [16] to unify general relativity and gauge symmetries. This realization naturally embeds the Standard Model and extends its fermion content by two heavy particles with opposite electromagnetic and Z-boson charges. Having no other gauge charges of the Standard model, these particles (AC-fermions) behave as heavy stable leptons with charges $-2e$ and $+2e$, called here A and C, respectively. The mass of the AC-fermions has a "geometrical" origin and is not related to the Higgs mechanism. Here "geometrical" does not refer to the Planck mass, but is attributed to the fact, that massive particles constitute an essential contribution to the whole geometric framework of noncommutative geometry. In the absence of AC-fermion mixing with light fermions, the AC-fermions can be absolutely stable. Such absolute stability naturally follows from strict conservation of the additional $U(1)$ gauge charge, which we call y-charge, ascribed to AC-leptons, and we explore this possibility in the present paper.

If the AC-leptons A and C have equal and opposite sign of y-charges, strict conservation of the y-charge does not prevent the generation of A and C excess; the excess of A being equal to the excess of C, as required in the further cosmological treatment.

AC-fermions are sterile relative to $SU(2)$ electro-weak interaction, and do not contribute to the standard model parameters.

Being absolutely stable, primordial heavy AC-leptons should be present in modern matter [50].

In the model [15] the properties of heavy AC-fermions are fixed by the almost-commutative geometry and the physical postulates given in [17]. The freedom resides in the choice of the hyper-charge and the masses. According to this model negatively charged A^{--} and positively charged C^{++} are stable and may form a neutral most probable and stable (while being evanescent) (AC) "atom". The AC-gas of such "atoms" is an ideal candidate [14, 15] for a very new and fascinating dark matter (like it was tera-helium gas in [12, 13]); because of their peculiar WIMP-like interaction with matter they may also rule the stages of gravitational clustering in early matter dominated epochs, creating first gravity seeds for galaxy formation.

However, in analogy to D, ^{3}He and Li relics that are the intermediate catalyzers of ^{4}He formation in the Standard Big Bang Nucleosynthesis (SBBN) and are important cosmological tracers of this process, the AC-lepton relics from intermediate stages of a multi-step process towards a final (AC) formation must survive with high abundance of *visible* relics in the present Universe. We enlisted, revealed and classified such tracers, their birth place and history up to now in [14].

We found in [14] that (eeC^{++}) should be here to remain among us and its abundance should be strongly reduced in terrestrial matter to satisfy known severe bounds on anomalous helium. This reduction is catalyzed by relic neutral OLe-helium (named so from *O-Le*pton- *helium*) $(^4He^{++}A^{--})$, because the primordial component of free anion-like AC-leptons A^{--} are mostly trapped in the first three minutes into this puzzling *OLe-helium* "atom" $(^4He^{++}A^{--})$ with nuclear interaction cross section, which provides anywhere an eventual later (AC) binding. This surprising catalyzer with screened Coulomb barrier can influence the chemical evolution of ordinary matter, but it turns out that the dominant process of OLe-helium interaction with nuclei is quasi-elastic and might not result in copious creation of anomalous isotopes. Inside dense matter objects (stars or planets) its recombination with (eeC^{++}) into (AC) atoms can provide a mechanism for the formation of dense (AC) objects. We have mentioned in [14] that most of the problems of AC-cosmology, related with possible fractionating of (eeC^{++}) and OLe-helium owing to the strong difference in their mobility in ordinary matter objects, can be avoided, if A and C possess an additional U(1) gauge charge (y-charge). The Coulomb-like attraction of y-charges prevents fractionating of anomalous helium and OLe-helium and makes them recombine effectively into (AC) atoms.

In the present paper we give some ideas on the exciting flavor of the unification based on almost commutative geometry and on the way it fixes the choice for the gauge symmetry group, underlying the AC-model [14, 15] (Section

I) and the properties of the AC-leptons A and C, predicted by it (Section II). We consider their evolution in the early Universe and notice (Section III) that in spite of the assumed excess of particles (A^{--} and C^{++}) the abundance of frozen out antiparticles ($\bar{A}^{++}$ and $\bar{C}^{--}$) is not negligible, as well as a significant fraction of A^{--} and C^{++} remains unbound, when AC recombination takes place and most of AC-leptons form (AC) atoms. This problem of an unavoidable over-abundance of by-products of "incomplete combustion" is unresolvable for models, assuming dark matter, composed of atoms, binding single charged particles, as it was revealed in [13] for the sinister Universe [12]. As soon as ^4He is formed in the Big Bang nucleosynthesis it captures all the free negatively charged heavy particles (Section III). If the charge of such particles is -1e (as it was the case for tera-electron in [12]) positively charged ion $(^4He^{++}E^-)^+$ puts up a Coulomb barrier for any successive decrease of abundance of the species, over-polluting by anomalous isotopes the modern Universe. The double negative charge of A^{--} in the considered AC-model [14] provides the binding with $^4He^{++}$ into a neutral Ole-helium state, which catalyzes in the first three minutes an effective binding into (AC) atoms and a complete annihilation of the antiparticles. Products of annihilation do not cause undesirable effects, neither in the CMB spectrum, nor in light element abundances. Due to early decoupling from the relativistic plasma y-photon background is suppressed and its contribution to the total density in the period of Big Bang Nucleosynthesis is compatible with observational constraints.

Still, though the CDM in the form of (AC) atoms is successfully formed, A^{--} (bound in OLe-helium) and C^{++} (forming anomalous helium atom (eeC^{++})) should be also present in the modern Universe and the abundance of primordial (eeC^{++}) is by up to *ten* orders of magnitude higher, than experimental upper limits on the anomalous helium abundance in terrestrial matter. This problem can be solved by OLe-helium catalyzed (AC) binding of (eeC^{++}) (Subsection III E), but different mobilities in matter of atomic interacting (eeC^{++}) and nuclear interacting (OHe) lead to the fractionating of these species, preventing an effective decrease of the anomalous helium abundance. We show that the $U(1)$ charge neutrality condition naturally prevents this fractionating, making (AC) binding sufficiently effective to suppress a terrestrial anomalous isotope abundance below the experimental upper limits.

However, though the (AC) binding is not accompanied by strong annihilation effects, as it was the case for 4th generation hadrons [10], gamma radiation from it inside large volume detectors should take place. We clarify the astrophysical uncertainties in estimation of the expected effect.

In this way AC-cosmology escapes most of the troubles, revealed for other cosmological scenarios with stable heavy charged particles [10, 13] and provides a realistic scenario for composite dark matter in the form of evanescent atoms, composed by heavy stable electrically charged particles, bearing the source of invisible light.

We give technical details for the approach to the particle theory, based on almost-commutative geometry, in Appendices 1-5.

I. A FLAVOR OF ALMOST-COMMUTATIVE GEOMETRY

In the last few years several approaches to include the idea of noncommutative spaces into physics have been established. One of the most promising and mathematically elaborated is Alain Connes *noncommutative geometry* [16] where the main idea is to translate the usual notions of manifolds and differential calculus into an algebraic language. Here we will mainly focus on the motivations why noncommutative geometry is a novel point of view of space-time, worthwhile to be considered by theoretical physics. We will furthermore try to give a glimpse on the main mathematical notions (for computational details see appendices 1 to 4), but refer to [16] and [18] for a thorough mathematical treatment and to [19] and [20] for its application to the standard model of particle physics.

Noncommutative geometry has its roots in quantum mechanics and goes back to Heisenberg [51] or even Riemann [21]. In the spirit of quantum mechanics it seems natural that space-time itself should be equipped with an uncertainty. The coordinate functions of space-time should be replaced by a suitable set of operators, acting on some Hilbert space with the dynamics defined by a Dirac operator. The choice of a relativistic operator is clear since the theory ought to be Lorentz invariant. As for the Dirac operator, in favor of the Klein-Gordon operator, matter is built from Fermions and so the Dirac operator is privileged. This approach, now known as noncommutative geometry, has been worked out by Alain Connes [16]. He started out on this field to find a generalized understanding to cope with mathematical objects that seemed geometrical, yet escaped the standard approaches. His work has its predecessors in Gelfand and Naimark [22], who stated that the topology of a manifold is encoded in the algebra of complex valued functions over the manifold. Connes extended this theorem and translated the whole set of geometric data into an algebraic language. The points of the manifold are replaced by the pure states of an algebra, which, inspired by quantum mechanics, acts on a Hilbert space. With help of a Dirac operator acting as well on the Hilbert space, Connes formulated a set of axioms which allows to recover the geometrical data of the manifold. These three items, the algebra, the Hilbert space and the Dirac operator are called a spectral triple. But it should be noted that the set of manifolds, i.e. space-times, which allow to be described by a spectral triple is limited. These manifolds have to be Riemannian, i.e. of Euclidean signature, and they have to admit a spin structure, which is not true for any manifold. The second condition presents

no drawback since space-time falls exactly into this class. But asking the manifold to be Euclidean, whereas special relativity requires a Lorentzian signature, poses a problem, which is still open. Nevertheless one can argue, along the line of Euclidean quantum field theory that this can be cured by Wick rotations afterwards.

A strong point in favor of the spectral triple approach is, as the name noncommutative geometry already implies, that the whole formulation is independent of the commutativity of the algebra. So even when the algebra is noncommutative it is possible to define a geometry in a consistent way. But then the geometry gets equipped with an uncertainty relation, just as in quantum mechanics. With this generalization comes a greater freedom to unify the two basic symmetries of nature, namely the diffeomorphism invariance (= invariance under general coordinate transformations) of general relativity and the local gauge invariance of the standard model of particle physics. In the case of ordinary manifolds theorems by O'Raifeartaigh [23], Coleman and Mandula [24] as well as the theorem of Mather [25] prohibit such a unification (for details see [16]).

The standard model can be constructed as a classical gauge theory which describes the known elementary particles by means of the symmetries they obey, together with the electro-weak and the strong force. In contrast to general relativity, this classical field theory allows to pass over to a quantum field theory. All elementary particles are fermions and the forces acting between them are mediated by bosons. The symmetries of the theory are compact Lie groups, for the standard model of particle physics the underlying symmetry goup is $U(1) \times SU(2) \times SU(3)$. Fermions are Dirac spinors, placed in multiplets which are representations of the symmetry groups. A peculiar feature of the standard model is that fermions are chiral. This poses a serious problem, since mass terms mixing left- and right-handed states would explicitly break the symmetry. To circumvent this an extra boson, the Higgs boson, has to be introduced. In the widely used formulation of the standard model this Higgs mechanism has to be introduced by hand. All the non-gravitational forces and all known matter is described in a unified way. But it is not possible to unify it on the footing of differential geometry with general relativity. The problem is that no manifold exists, which has general coordinate transformations and a compact Lie group as its diffeomorphism group. But here the power of noncommutative geometry comes in.

The first observation is that the general coordinate transformations of a manifold correspond to the automorphisms [52] of the algebra of complex valued functions over the manifold. Chamseddine and Connes [19] discovered that it is possible to define an action, called the *spectral action*, to give space-time in the setting of spectral triples a dynamics, just as the Einstein-Hilbert action for general relativity. This spectral action is given by the number of eigenvalues of the Dirac operator up to a cut-off. It is most remarkable that this action reproduces the Einstein-Hilbert action in the limit of high eigenvalues of the Dirac operator. The crucial observation is now that in contrast to the diffeomorphisms of a manifold, the automorphisms of an algebra allow to be extended to include compact Lie groups. These are the automorphisms of matrix algebras. And since the whole notion of a spectral triple is independent of the commutativity of the algebra, it is possible to combine the algebra of functions over the space-time manifold with an algebra being the sum of simple matrix algebras by tensorising. These combined function-matrix geometries are called *almost-commutative* geometries. The part of the spectral triple based on the matrix algebra is often called the *finite* or *internal part*. Indeed, they contain an infinite number of commutative degrees of freedom plus a finite number of noncommutative ones. The former are outer, the latter are inner automorphisms.

To see how the Higgs scalar, gauge potentials and gravity emerge one starts out with an almost-commutative spectral triple over a flat manifold M. The corresponding algebra of complex valued functions over the manifold will be $\mathcal{A}_R$ (where the subscript R stands for *Riemannian* [53]), the Hilbert space $\mathcal{H}_R$ is the Hilbert space of Dirac spinors and the Dirac operator is simply the flat Dirac operator $\partial\!\!\!/$. As mentioned above, the automorphisms of the algebra $\mathcal{A}_R$ coincide with the diffeomorphisms, i.e. the general coordinate transformations, of the underlying manifold, $\mathrm{Aut}(\mathcal{A}_R) = \mathrm{Diff}(M)$. To render this function algebra noncommutative, a matrix algebra $\mathcal{A}_f$ (where the subscript f stands for *finite*) is chosen. The exact form of this matrix algebra is of no importance for the moment (as long as its size is at least two). The Hilbert space $\mathcal{H}_f$ is finite dimensional and the Dirac operator $\mathcal{D}_f$ is a complex valued matrix. For the detailed form of the internal Dirac operator see Appendix 1.

It is a pleasant feature of spectral triples that the tensor product of two spectral triples is again a spectral triple. So building the tensor product one finds for the algebra and the Hilbert space of the almost commutative geometry

$$\mathcal{A}_{AC} = \mathcal{A}_R \otimes \mathcal{A}_f, \quad \mathcal{H}_{AC} = \mathcal{H}_R \otimes \mathcal{H}_f. \tag{1}$$

The Dirac operator needs a little bit more care to comply with the axioms for spectral triples. It is given by

$$\mathcal{D}_{AC} = \partial\!\!\!/ \otimes 1_f + \gamma^5 \otimes \mathcal{D}_f, \tag{2}$$

where 1_f is a unity matrix whose size is the size of the finite Dirac operator $\mathcal{D}_f$ and γ^5 is constructed in the standard way from the Dirac gamma matrices. The automorphism group of the almost-commutative algebra $\mathcal{A}_{AC}$ is the semi direct product of the diffeomorphisms of the underlying manifolds and the gauged automorphisms of the matrix algebra. For example, with the matrix algebra $\mathcal{A}_f = M_2(\mathbb{C})$ one would have the gauged unitary group $SU(2)$ in the automorphism group. This is exactly the desired form for a symmetry group.

Now these automorphisms

$$Aut(\mathcal{A}_{AC}) = Aut(\mathcal{A}_R) \ltimes Aut(\mathcal{A}_f) \ni (\sigma_R, \sigma_f) \tag{3}$$

have to be lifted (=represented) to the Hilbert space $\mathcal{H}_{AC}$. This is necessary to let them act on the Fermions as well as to fluctuate or gauge the Dirac operator. It is achieved by the lift $L(\sigma_R, \sigma_f)$ which is defined via the representation of the algebra on the Hilbert space. For details see Appendix 4.

For the moment the Dirac operator $\mathcal{D}_{AC}$ consists of the Dirac operator on a flat manifold and a complex valued matrix. Now, to bring in the Higgs scalar, the gauge potentials and gravity the Dirac operator has to be fluctuated or gauged with the automorphisms (3)

$$
\begin{aligned}
{}^f\mathcal{D}_{AC} \quad &:= L(\sigma_R, \sigma_f)\mathcal{D}_{AC}L(\sigma_R, \sigma_f)^{-1} \\
&= L(\sigma_R, \sigma_f)(\partial\!\!\!/ \otimes 1_f)L(\sigma_R, \sigma_f)^{-1} + L(\sigma_R, \sigma_f)(\gamma^5 \otimes \mathcal{D}_f)L(\sigma_R, \sigma_f)^{-1} \\
&= \partial\!\!\!/_{cov.} + \gamma^5 \otimes L(\sigma_f)\mathcal{D}_fL(\sigma_f)^{-1} = \partial\!\!\!/_{cov.} + \gamma^5 \otimes {}^f\mathcal{D}_f.
\end{aligned}
\tag{4}
$$

In the last step it turns out that $\partial\!\!\!/_{cov.} := L(\sigma_R, \sigma_f)(\partial\!\!\!/ \otimes 1_f)L(\sigma_R, \sigma_f)^{-1}$ is indeed the covariant Dirac operator on a curved space time, when the appearing gauge potentials have been promoted to arbitrary functions, i.e. after applying Einstein's equivalence principle (for details see [19]). $\partial\!\!\!/_{cov.}$ has automatically the correct representation of the gauge potentials on the Hilbert space of Fermion multiplets. The gauge potentials thus emerge from the usual Dirac operator acting on the gauged automorphisms of the inner algebra.

As for the Higgs scalar, it is identified with ${}^f\mathcal{D}_f := L(\sigma_f)\mathcal{D}_fL(\sigma_f)^{-1}$. Here the commutative automorphisms being the diffeomorphisms σ_R of the manifold drop out, since they commute with the matrix $\mathcal{D}_f$. This is not true for the gauged automorphisms σ_f since they are matrices themselves.

From the gauged Dirac operator ${}^f\mathcal{D}_{AC}$ the spectral action is calculated via a heat-kernel expension to be the Einstein-Hilbert action plus the Yang-Mills-Higgs action. The Higgs potential in its well known quartic form is a result of this calculation. It should be pointed out that the heat-kernel expansion is performed up to a cut-off and so the obtained Einstein-Hilbert action and Yang-Mills-Higgs action should be considered as effective actions. The details of the calculation of the spectral action goes beyond the scope of this publication and we refer again to [19] for a detailed account. For the internal part ${}^f\mathcal{D}_f$ of the gauged Dirac operator ${}^f\mathcal{D}_{AC}$ the spectral action gives exactly the Higgs potential

$$V({}^f\mathcal{D}_f) = \lambda \, \text{tr}\big[({}^f\mathcal{D}_f)^4\big] - \tfrac{\mu^2}{2} \, \text{tr}\big[({}^f\mathcal{D}_f)^2\big], \tag{5}$$

where λ and μ are positive constants, as well as the kinetic term for the Higgs potential. To determine the a sensible value for the cut-off in the heat-kernel expansion, it is instructive to note, that at the cut-off the couplings of the non-abelian gauge groups and the coupling λ of the Higgs potential are closely tied together.

Choosing as matrix algebra $\mathcal{A}_f = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, where $\mathbb{H}$ are the quaternions, one recovers with a suitable choice for the Hilbert space that the spectral action reproduces the Einstein-Hilbert action and the Yang-Mills-Higgs action of the standard model. The cut-off is then fixed to be at the energy where the coupling g_2 of the weak group $SU(2)_L$ and the coupling g_3 of the colour group $SU(3)_C$ become equal ($\sim 10^{17}$GeV). At the cut-off these two couplings and the Higgs coupling λ are related as

$$g_3^2 = g_2^2 = 3\lambda. \tag{6}$$

Assuming a great dessert up to the cut-off, this relation allows to let the Higgs coupling run back to lower energies and to calculate the Higgs mass a. A detailed calculation can be found in [20] and gives a Higgs mass of $m_{Higgs} = 175.4\pm4.7$ GeV, where the uncertainty is due to the uncertainty in the top-quark mass.

Recapitulating, the Higgs scalar together with its potential emerge naturally as the "Einstein-Hilbert action" in the noncommutative part of the algebra. Here it has become possible for the first time to give the Higgs scalar a geometrical interpretation. In the almost-commutative setting it plays at the same time the rôle of the metric in the finite part of the geometry as well as that of the fermionic mass matrix.

One may interpret an almost-commutative geometry as a kind of Kaluza-Klein theory, where the extra dimensions are discrete. These extra dimensions are produced by the matrix algebra and they provide for extra degrees of freedom without being visible. Furthermore the Yang-Mills-Higgs action can be viewed in the almost-commutative setting as the gravitational action, or Einstein-Hilbert analogue for the "discrete part" of space-time. From this point of view the gauge bosons, i.e. the Higgs boson, the Yang-Mills bosons and the graviton form a unified "super-multiplet". But of course space-time in the classical sense ceases to exist in noncommutative geometry, just as there is no classical phase space in quantum mechanics. The space-time has been replaced by operators and extended by discrete extra dimensions.

Extensions of the Standard Model

Yang-Mills-Higgs
Models

● = Standard Model   ○ = AC-Model

☐ = Noncommutative Ext.   ☐ = Almost-Commutative Ext.

☐ = Supersymmetric Ext.   ☐ = Left-Right-Symm. Ext.
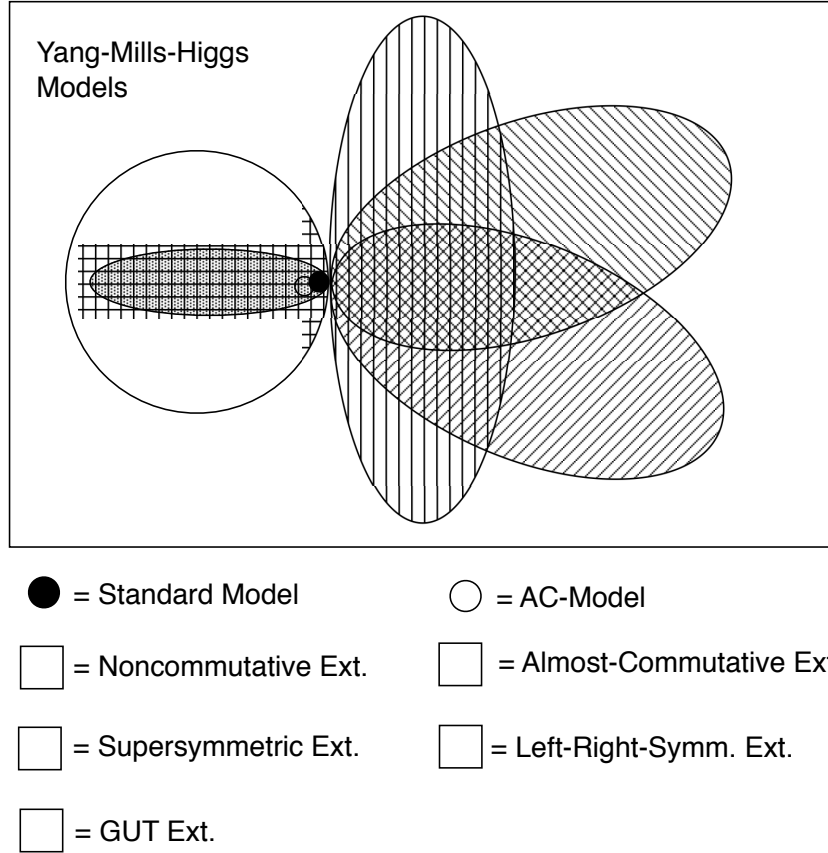
☐ = GUT Ext.

FIG. 1: Yang-Mills-Higgs models extending the Standard Model

The immediate question that arises is: Which kind of Yang-Mills-Higgs theory may fit into the frame work of almost-commutative geometry? The set of all Yang-Mills-Higgs theories is depicted in figure 1. One sees that left-right symmetric, grand unified and supersymmetric theories do not belong to the elected group of noncommutative models. But, as mentioned above, the standard model, resulting from an almost-commutative geometry, as well as the AC-model, do.

One of the main tasks of the present research in almost-commutative geometry is to clarify the structure of this restricted sub-set of Yang-Mills-Higgs theories that originate from spectral triples. Since this is still an unscalable challenge it is necessary to adopt a minimal approach. Imposing certain constraints which are gathered from different areas reaching from Riemannian geometry over high energy physics to quantum field theory and starting out with only up to four summands in the matrix algebra part of the almost commutative geometry, one can give a classification from the particle physicist's point of view. As it is custom in particle physics, space-time curvature will be neglected. Nonetheless the Riemannian part of the spectral triple plays a crucial role in the spectral action, introducing derivatives and thus the gauge bosons. Setting the curvature to zero when the Einstein-Hilbert and Yang-Mills-Higgs action have been obtained from the spectral action leaves the Yang-Mills-Higgs action. With respect to this part of the spectral action the classification will be done. As a consequence only the finite matrix algebra part of the spectral triple has to be classified since only the internal Dirac operator enters into the Higgs scalar, as was shown above. The minimum of the Higgs potential is the mass matrix of the fermions.

This classification proceeds in two steps. First all the possible finite spectral triples, with a given number of summands of simple matrix algebras, have to be found. This classification of finite spectral triples has been done in the most general setting by Paschke, Sitarz [26] and Krajewski [27]. To visualize a finite spectral triple Krajewski introduced a diagrammatic notion, *Krajewski diagrams*, which encode all the algebraic data of a spectral triple. For a more detailed account see Appendix 3. If one imposes now as a first condition that the spectral triple be irreducible, i.e. that the finite Hilbert space be as small as possible, one is led to the notion of a minimal Krajewski diagram. For a given number of algebras, the algebra representation on the Hilbert space and the possible Dirac operators

are encoded in these diagrams by arrows connecting two sub-representations. Finding the minimal diagrams via this diagrammatic approach is very convenient and quite simple for up to two summands in the matrix algebra. In this case only a handful of diagrams exist and it is difficult to miss a diagram. But with three and more algebras the task quickly becomes intractable. For three algebras it may be done by hand, but one risks to overlook some diagrams. It is thus fortunate that the diagrammatic treatment allows to translate the algebraic problem of finding spectral triples into the combinatorial problem of finding minimal Krajewski diagrams. This can then be put into a computer program. Still the problem is quite involved and the algorithm to find minimal Krajewski diagrams needs a lot of care. Furthermore the number of possible Krajewski diagrams increases rapidly with the number of summands of matrix algebras and reaches the maximal capacity of an up-to-date personal computer at five summands.

Nonetheless it is possible to find a Krajewski diagram with six summands in the matrix algebra which is in concordance with the physical requirements presented below. It is the aim of this paper to evaluate its impact on the dark matter problem in cosmology.

If one has found the minimal Krajewski diagrams the second major step follows. From each Krajewski diagram all the possible spectral triples have to be extracted. These are then analyzed with respect to the following heteroclitic criteria:

- For simplicity and in view of the minimal approach the spectral triple should be irreducible. This means simply that the Hilbert space cannot be reduced while still obeying all the axioms of a spectral triple.

- The spectral triple should be non-degenerate, which means that the fermion masses should be non-degenerate, up to the inevitable degeneracies which are left and right, particle and antiparticle and a degeneracy due to a color. This condition has its origin in perturbative quantum field theory and asserts that the possible mass equalities are stable under renormalization flow.

- Another criterion also stemming from quantum field theory is that the Yang-Mills-Higgs models should be free of Yang-Mills anomalies. In hope of a possible unified quantum theory of all forces, including gravity, it is also demanded that the models be free of mixed gravitational anomalies.

- From particle phenomenology originates the condition that the representation of the little group has to be complex in each fermion multiplet, in order to distinguish particles from antiparticles.

- The last item is the requirement that massless fermions should be neutral under the little group. This is of course motivated by the Lorentz force.

Now the Higgs potential has to be minimized and the resulting models have to be compared with the above list of criteria. If a model fits all the points of the list it may be considered of physical importance, otherwise it will be discarded.

II. THE PARTICLE MODEL

Among the possible almost-commutative Yang-Mills-Higgs models is the standard model of particle physics with one generation of quarks and leptons, for details we refer to [20]. It could furthermore be shown, [17, 28, 29, 30], that the standard model takes a most prominent position among these Yang-Mills-Higgs models.

But the classification of almost-commutative geometries also allows to go beyond the standard model in a coherent way. Here heavy use is made of Krajewski diagrams [27], which allow to visualize the structure of almost-commutative geometries. The particle model analyzed in the present publication is an extension of the AC-lepton model presented in [15] which was analyzed with respect to its cosmological implications in [14]. It is remarkable to note that the almost-commutative geometry of the basic AC-lepton model, which builds on the internal algebra

$$\mathcal{A} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}, \tag{7}$$

allows to be enlarged to

$$\mathcal{A} = \mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \tag{8}$$

which produces through the so called centrally extended lift, for details see [31], a second $U(1)$ gauge group in addition to the standard model hypercharge group $U_Y(1)$. In spirit with the previous nomenclature this group will be called $U_{AC}(1)$, where AC stands again for almost-commutative. Choosing the central charges to reproduce the standard model and the AC-particles with the correct electric charges, a minimal extension consists in coupling this new gauge group in the Lagrangian only to the AC-fermions. For a detailed derivation from the corresponding Krajewski diagram

to the Lagrangian of the model we refer to Appendix 5. The AC-particles do not participate in the Higgs mechanism and consequently the AC-gauge group stays unbroken:

$$U_Y(1) \times SU_L(2) \times SU_c(3) \times U_{AC}(1) \longrightarrow U_{em}(1) \times SU_c(3) \times U_{AC}(1)$$

The Lagrangian of the model consists of the usual standard model Lagrangian, the Lagrangian for the AC-particles and the new term for the AC-gauge potential. We shall only give the two new parts of the Lagrangian for the AC-fermion spinors ψ_A and ψ_C and the AC-gauge curvature $\tilde{B}_{\mu\nu}$:

$$\begin{aligned}
\mathcal{L}_{AC} &= i\psi^*_{AL}D_A\psi_{AL} + i\psi^*_{AR}D_A\psi_{AR} + m_A\psi^*_{AL}\psi_{AR} + m_A\psi^*_{AR}\psi_{AL} \\
&+ i\psi^*_{CL}D_C\psi_{CL} + i\psi^*_{CR}D_C\psi_{CR} + m_C\psi^*_{CL}\psi_{CR} + m_C\psi^*_{CR}\psi_{CL} \\
&- \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}.
\end{aligned}$$

The covariant derivatives $D_{A/C}$ and the gauge curvature are given by

$$\begin{aligned}
D_{A/C} &= \gamma^\mu\partial_\mu + \frac{i}{2}g'Y_{A/C}\gamma^\mu B_\mu + \frac{i}{2}g_{AC}\tilde{Y}_{A/C}\gamma^\mu\tilde{B}_\mu \\
&= \gamma^\mu\partial_\mu + \frac{i}{2}e Y_{A/C}\gamma^\mu A_\mu - \frac{i}{2}g'\sin\theta_w Y_{A/C}\gamma^\mu Z_\mu + \frac{i}{2}g_{AC}\tilde{Y}_{A/C}\gamma^\mu\tilde{B}_\mu,
\end{aligned}$$

and

$$\tilde{B}_{\mu\nu} = \partial_\mu\tilde{B}_\nu - \partial_\nu\tilde{B}_\mu \tag{9}$$

As has been pointed out in [14], the electric charge of the AC-leptons has to be $Q_{em} = \pm 2e$, where e is the electric charge of the electron. Otherwise unwanted forms of OLe-Helium ions would appear. This requires $Y_{A/C} = \mp 2$. For simplicity the AC-hyper charge is also chosen to be $\tilde{Y}_{A/C} = \mp 2$, but it cannot be fixed by almost-commutative geometry. This also applies to the AC-coupling g_{AC} which has to be fixed by experiment. If the coupling is chosen small enough, the AC-fermions will exhibit a supplementary long range force with a Coulomb like behavior. The corresponding necessarily massless "photons" will be called y-photons. Implications and effects on the high energy physics of the standard model will not be considered in this paper, but there may be detectable effects due to interactions between AC-fermions and standard model particles on loop level.

Indeed, loop diagrams with virtual A and C pairs induce mixing between y-photon and ordinary gauge bosons ($y - \gamma$ and $y - Z$). Due to this mixing ordinary particles acquire new long range (y) interaction, which, however, can be masked in the electro-neutral matter.

The masses of the standard model fermions are obtained by minimizing the Higgs potential. It turns out that the masses m_A and m_C of the new fermions do not feel the fluctuations of the Dirac operator and are thus Dirac masses which do not stem from the Higgs mechanism but have a purely geometrical origin. This is due to the necessarily vector like coupling of the gauge group induced by the lift of the automorphisms. Consequently these Dirac masses do not break gauge invariance. The mass scale will later be fixed on cosmological grounds.

III. THE COSMOLOGICAL MODEL

The model [14, 15] admits that in the early Universe a charge asymmetry of AC-fermions can be generated, as it is the case for ordinary baryons, so that an A and a C excess saturates the modern dark matter density, dominantly in the form of (AC) atoms. For the baryon excess $\eta_b = n_{b\,mod}/n_{\gamma\,mod} = 6 \cdot 10^{-10}$ it gives an AC-excess

$$\eta_A = n_{A\,mod}/n_{\gamma\,mod} = \eta_C = n_{C\,mod}/n_{\gamma\,mod} = 3 \cdot 10^{-11}(\frac{100\,\text{GeV}}{M}), \tag{10}$$

where $M = m_A + m_C$ is the sum of the masses of A and C. Following [12, 13, 14], it is convenient to relate the baryon $\Omega_b = 0.044$ and the AC-lepton densities $\Omega_{CDM} = 0.224$ with the entropy density s and to introduce $r_b = n_b/s$ and $r_A = r_C = n_A/s = n_C/s$. Taking into account that $s_{mod} = 7.04 \cdot n_{\gamma\,mod}$, one obtains $r_b \sim 8 \cdot 10^{-11}$ and

$$r_A = r_C = 4 \cdot 10^{-12}(\frac{100\,\text{GeV}}{M}). \tag{11}$$

We'll further assume that $m_A = m_C = M/2 = m$, so the AC -fermion excess Eq.(11) is given by

$$\kappa_A = \kappa_C = r_A - r_{\bar{A}} = r_C - r_{\bar{C}} = 2 \cdot 10^{-12}(\frac{100\,\text{GeV}}{m}) = 2 \cdot 10^{-12}/S_2, \tag{12}$$

where $S_2 = m/100\,\text{GeV}$.

A. Chronological cornerstones of the AC-Universe

After the generation of AC-lepton asymmetry in chronological order the thermal history of AC-matter follows the trend, which we have thoroughly studied in [14] for $m_A = m_C = m = 100S_2$ GeV. Therefore we briefly outline here this trend, specifying in more details the effects of the y-interaction

1) $10^{-10}S_2^{-2}$ s $\leq t \leq 6 \cdot 10^{-8}S_2^{-2}$ s at $m \geq T \geq T_f = m/31 \approx 3S_2$ GeV. AC-lepton pair $A\bar{A}$ and $C\bar{C}$ annihilation and freezing out take place. At $S_5 \leq 5$ frozen out concentration of antiparticles is exponentially suppressed, while the concentration of A and C tends to the value of their excess (11). For larger m the abundance of frozen out AC-lepton pairs is not suppressed in spite of AC-lepton excess and antiparticles survive until later stages.

2) $1.5 \cdot 10^{-4}S_2^{-2}k_y^{-4}$ s $\leq t \leq 1.3 \cdot 10^{-1}S_2^{-2}k_y^{-4}$ s at $I_{AC} \approx 80S_2k_y^2$ MeV $\geq T \geq I_{AC}/30$. In this period recombination of negatively charged AC-leptons A^{--} with positively charged C^{++}-leptons can lead to the formation of AC-lepton "atoms" (AC) with potential energy $I_{AC} = Z_A^2Z_C^2\alpha^2k_y^2m \approx 80S_2k_y^2$ MeV $(Z_A = Z_C = 2$ and $k_y = (1 + \alpha_y/(Z_AZ_C\alpha))/2 \approx 1$ for $\alpha_y \sim 1/30)$. Together with neutral (AC)-"atoms" free charged A^{--} and C^{++} are also left, being the dominant form of AC-matter at $S_2 > 6$.

3) $t \sim 1.5 \cdot 10^{-4}S_2^{-2}k_y^{-4}$ s at $T \sim I_A = I_C = 80S_2k_y^2$ MeV. The temperature corresponds to the binding energy $I_A = I_C = Z_A^4\alpha^2k_y^2m = Z_C^4\alpha^2k_y^2m \approx 80S_2k_y^2$ MeV $(Z_A = Z_C = 2)$ of twin AC-positronium "atoms" $(A^{--}\bar{A}^{++})$ and $(C^{++}\bar{C}^{--})$, in which $\bar{A}^{++}$ and $\bar{C}^{--}$ annihilate. At large m this annihilation is not at all effective to reduce the $A\bar{A}$ and $C\bar{C}$ pairs abundance. These pairs are eliminated in the course of the successive evolution of AC-matter.

4) 100 s $\leq t \leq 300$ s at 100 keV $\geq T \geq I_{OHe}/27 \approx 60$ keV, where $I_{OHe} = Z_{He}^2Z_A^2\alpha^2m_{He}/2 = 1.6$ MeV is the ionization potential of a $(^4He^{++}A^{--})$ "atom". Helium ^4He is formed in the Standard Big Bang Nucleosynthesis and virtually all free A^{--} are trapped by ^4He in OLe-helium $(^4He^{++}A^{--})$. Note that in the period 100 keV $\leq T \leq 1.6$ MeV helium ^4He is not formed, therefore it is only after *the first three minutes*, when $(^4He^{++}A^{--})$ trapping of A^{--} can take place. Being formed, OLe-helium catalyzes the binding of free C^{++} with its constituent A^{--} into (AC)-"atoms". In this period free $\bar{C}^{--}$ are also captured by ^4He. At large m effects of $(A^{--}\bar{A}^{++})$ and $(C^{++}\bar{C}^{--})$ annihilation, catalyzed by OLe-helium, do not cause any contradictions with observations.

The presence of new relativistic species - a gas of primordial y-photons - does not influence the light element abundances, since the y-photons decouple at $T < T_f(Z^2\alpha/\alpha_y)$ from the cosmological plasma after AC-lepton pairs are frozen out at $T_f = m/30 \approx 3S_2$ GeV. Here α_y is the fine structure constant of the y-interaction and $Z = 2$. Therefore the contribution of y-photons into the total number of relativistic species in the period of SBBN is suppressed.

B. OLe-helium in the SBBN

OLe-helium looks like an α particle with shielded electric charge. It can closely approach nuclei due to the absence of a Coulomb barrier. On that reason it seems that in the presence of OLe-helium the character of SBBN processes should change drastically. However, this change might be not so dramatic [11].

In fact, the size of OLe-helium is of the order of the size of ^4He and for a nucleus Z with electric charge $Z > 2$ the size of the Bohr orbit for an $A^{--}Z$ ion is less than the size of nucleus Z. This means that while binding with a heavy nucleus A^{--} penetrates it and effectively interacts with a part of the nucleus with a size less than the corresponding Bohr orbit. This size corresponds to the size of ^4He, making OLe-helium the most bound $A^{--}Z$-atomic state. It favors the picture, according to which OLe-helium collision with a nucleus, results in the formation of OLe-helium and the whole process looks like an elastic collision.

The interaction of the ^4He component of $(^4He^{++}A^{--})$ with a $_Z^AQ$ nucleus can lead to a nuclear transformation due to the reaction

$$_Z^AQ + (^4He^{++}A^{--}) \rightarrow _{Z+2}^{A+4}Q + A^{--}, \tag{13}$$

provided that the masses of the initial and final nuclei satisfy the energy condition

$$M(A,Z) + M(4,2) - I_{OHe} > M(A+4, Z+2), \tag{14}$$

where $I_{OHe} = 1.6$ MeV is the binding energy of OLe-helium and $M(4,2)$ is the mass of the helium nucleus.

The final nucleus is formed in the excited $[\alpha, M(A,Z)]$ state, which can rapidly experience an α-decay, giving rise to a (OHe) regeneration and to an effective quasi-elastic process of (OHe)-nucleus scattering. It leads to a possible suppression of the nuclear transformation (13).

The condition (14) is not valid for stable nuclei participating in reactions of the SBBN. However, unstable tritium ^3H, produced in SBBN and surviving 12.3 years after it, can react with OLe-helium, forming ^7Li in process $^3H + (^4HeA^{--}) \rightarrow ^7Li + A^{--}$. The Alexium A^{--}, released in this process, is captured by ^4He and regenerates OLe-helium, while ^7Li reacts with OLe-helium, forming ^{11}B etc. After ^{39}K the chain of transformations starts to create

unstable isotopes and gives rise to an extensive tree of transitions along the table of nuclides. This set of processes involves the fraction of baryons of the order of SBBN tritium abundance ($^3H/H \sim 10^{-7}$) and since it does not stop on lithium, but goes further to nuclides which are observed now with much higher abundance, it can not be excluded by a simple argument. This picture opens a new path of chemical evolution of matter on the pre-galactic stage and needs self-consistent consideration within a complete network of nuclear processes.

C. OLe-helium catalysis of (AC) binding

The process of C^{++} capture by the (OHe) atom looks as follows [14]. Being in thermal equilibrium with the plasma, free C^{++} have momentum $k = \sqrt{2Tm_C}$. If their wavelength is much smaller than the size of the (OHe) atom, they can penetrate the atom and bind with A^{--}, expelling He from it. The rate of this process is determined by the size of the (OHe) atoms and is given in [14] as

$$\langle \sigma v \rangle_0 \sim \pi R_{OHe}^2 \sim \frac{\pi}{(\bar{\alpha} m_{He})^2} = \frac{\pi}{2 I_{OHe} m_{He}} \approx 3 \cdot 10^{-15} \frac{cm^3}{s}.$$

Here $\bar{\alpha} = Z_A Z_{He} \alpha$. At $T < T_a = \bar{\alpha}^2 m_{He} \frac{m_{He}}{2m_C} = \frac{I_{OHe} m_{He}}{m_C} = 4 \cdot 10^{-2} I_{OHe} / S_2$ the wavelength of C^{++}, λ, exceeds the size of (OHe) and the rate of (OHe) catalysis is suppressed by a factor $(R_{OHe}/\lambda)^3 = (T/T_a)^{3/2}$ and is given by

$$\langle \sigma v \rangle_{cat} (T < T_a) = \langle \sigma v \rangle_0 \cdot (T/T_a)^{3/2}.$$

In the presence of the y-interaction both OLe-helium and C^{++} are y-charged and for slow charged particles a Coulomb-like factor of the "Sakharov enhancement" [2] should be added in these expressions

$$C_y = \frac{2\pi \alpha_y / v}{1 - \exp(-2\pi \alpha_y / v)},$$

where $v = \sqrt{2T/m}$ is relative velocity. It results in

$$\langle \sigma v \rangle_0 = \pi R_{OHe}^2 \cdot 2\pi \alpha_y \cdot (m/2T)^{1/2} = \frac{\alpha_y}{\bar{\alpha}} \frac{\pi^2}{I_{OHe} m_{He}} \cdot \left(\frac{m}{m_{He}}\right)^{1/2} \frac{1}{x^{1/2}} \approx 10^{-13} \frac{\alpha_y}{1/30} \left(\frac{S_2}{x}\right)^{1/2} \frac{cm^3}{s}, \qquad (15)$$

where $x = T/I_{OHe}$. At $T < T_a$ the rate of OLe-helium catalysis is given by

$$\langle \sigma v \rangle_{cat} (T < T_a) = \langle \sigma v \rangle_0 \cdot (T/T_a)^{3/2} = \frac{\alpha_y}{\bar{\alpha}} \frac{\pi^2}{T_a m_{He}} \cdot \left(\frac{T}{T_a}\right) \approx 2 \cdot 10^{-19} \frac{\alpha_y}{1/30} S_2^2 \left(\frac{T}{300K}\right) \frac{cm^3}{s}. \qquad (16)$$

The "Coulomb-like" attraction of y-charges can lead to their radiative recombination. It can be described in the analogy to the process of free monopole-antimonopole annihilation considered in [32]. The potential energy of the Coulomb-like interaction between A and C exceeds their thermal energy T at the distance

$$d_0 = \frac{\alpha_y}{T}.$$

Following the classical solution of energy loss due to radiation, converting infinite motion to finite, free y-charges form bound systems at the impact parameter [10, 32]

$$a \approx (T/m)^{3/10} \cdot d_0. \qquad (17)$$

The rate of such a binding is then given by [10]

$$\langle \sigma v \rangle = \pi a^2 v \approx \pi \cdot (m/T)^{9/10} \cdot \left(\frac{\alpha_y}{m}\right)^2 \approx \qquad (18)$$

$$\approx 2 \cdot 10^{-12} \left(\frac{\alpha_y}{1/30}\right)^2 \left(\frac{300K}{T}\right)^{9/10} S_2^{-11/10} \frac{cm^3}{s}$$

The successive evolution of this highly excited atom-like bound system is determined by the loss of angular momentum owing to the y-radiation. The time scale for the fall into the center in this bound system, resulting in AC recombination, was estimated according to classical formula in [10, 33]

$$\tau = \frac{a^3}{64\pi} \cdot (\frac{m}{\alpha_y})^2 = \frac{\alpha_y}{64\pi} \cdot (\frac{m}{T})^{21/10} \cdot \frac{1}{m} \tag{19}$$

$$\approx 2 \cdot 10^{-4}(\frac{\alpha_y}{1/30})(\frac{300K}{T})^{21/10} S_2^{11/10} s.$$

As is easily seen from Eq.(19) this time scale of AC recombination $\tau \ll m/T^2 \ll m_{Pl}/T^2$ turns out to be much less than the cosmological time at which the bound system was formed.

The above classical description assumes $a = \alpha_y/(m^{3/10}T^{7/10}) \gg 1/(\alpha_y m)$ and is valid at $T \ll T_{rc} = m\alpha_y^{20/7} \approx 60\,\mathrm{MeV} S_2(\frac{\alpha_y}{1/30})^{20/7}$ [10, 13]. Since $T_{rc} \gg I_{OHe}$ effects of radiative recombination can also contribute AC-binding due to OLe-helium catalysis. However, the rate of this binding is dominated by (16) at

$$T \leq T_a(\frac{\alpha_y \bar\alpha^{6/5}}{\pi^2})^{10/19}(\frac{m_{He}}{m})^{11/19} \approx 100\,\mathrm{eV}(\frac{\alpha_y}{1/30})^{10/19} S_2^{-30/19} \tag{20}$$

and the radiative recombination becomes important only at a temperature much less, than in the period of cosmological OLe-helium catalysis.

At modest values of S_2 the abundance of primordial antiparticles is suppressed [14] and the abundance of free C, r_C, is equal to the abundance of A^{--}, trapped in the (OHe) atoms, $r_A = r_{OHe}$. Therefore a decrease of their concentration due to the OLe-helium catalysis of (AC) binding is determined by the equation

$$\frac{dr_C}{dx} = f_{1He} \langle \sigma v \rangle r_C r_{OHe}, \tag{21}$$

where $x = T/I_{OHe}$, $r_{OHe} = r_C$, $\langle \sigma v \rangle$ is given by Eqs.(15) at $T > T_a$ and (16) at $T < T_a$, $\bar\alpha = Z_A Z_{He}\alpha$ and

$$f_{1He} = \sqrt{\frac{\pi g_s^2}{45 g_\epsilon}} m_{Pl} I_{OHe} \approx m_{Pl} I_{OHe}.$$

The solution of Eq.(21) is given by

$$r_C = r_{OHe} = \frac{r_{C0}}{1 + r_{C0}J_{OHe}} \approx \frac{1}{J_{OHe}} \approx 7 \cdot 10^{-20}(\frac{1/30}{\alpha_y})/f(S_2).$$

Here

$$J_{OHe} = \int_0^{x_{fHe}} f_{1He} \langle \sigma v \rangle \, dx =$$

$$= \pi^2 \frac{\alpha_y}{\bar\alpha}(\frac{m_{Pl}}{2m_{He}})f(S_2) \approx 1.4 \cdot 10^{19}(\frac{\alpha_y}{1/30}) \cdot f(S_2), \tag{22}$$

$x_{fHe} = 1/27$ and the dependence on S_2 is described by the function $f(S_2) = 4(\frac{S_2}{1.08})^{1/2} - 3$ for $S_2 > 1.08$; $f(S_2) = (\frac{S_2}{1.08})^2$ for $S_2 < 1.08$.

At large $S_2 > 40$ the primordial abundance of antiparticles ($\bar A$ and $\bar C$) is not suppressed. OLe-helium catalyze in this case annihilation of these antiparticles through the formation of AC-positronium and it was shown in [14] that electromagnetic showers, induced by annihilation products can neither influence the light element abundance, nor cause observable distortions of the CMB spectrum.

Colexium C^{++} ions, which remain free after OLe-helium catalysis, are in thermal equilibrium due to their Coulomb scattering with matter plasma. At $T < T_{od} \approx 1\,\mathrm{keV}$ energy and momentum transfer due to nuclear interaction from baryons to OLe-helium is not effective $n_b \langle \sigma v \rangle (m_p/m_o)t < 1$. Here

$$\sigma \approx \sigma_{OHe} \sim \pi R_{OHe}^2 \approx 10^{-25}\,\mathrm{cm}^2. \tag{23}$$

and $v = \sqrt{2T/m_p}$ is baryon thermal velocity. Then OLe-helium gas decouples from plasma and radiation and must behave like a sparse component of dark matter. However, for a small window of parameters $1 \leq S_2 \leq 2$ at $T < (\frac{\alpha_y}{1/30})\frac{10\,\text{eV}}{S_2 f(S_2)^2}$ Coulomb-like scattering due to y interaction with C^{++} ions returns OLe-helium to thermal equilibrium with plasma and supports effective energy and momentum exchange between A and C components during all the pre-galactic stage.

5) $t \sim 2.5 \cdot 10^{11}$ s at $T \sim I_{He}/30 \approx 2eV$. Here $I_{He} = Z^2\alpha^2 m_e/2 = 54.4\,\text{eV}$ is the potential energy of an ordinary He atom. Free C^{++} with charge $Z = +2$ recombine with e^- and form anomalous helium atoms (eeC^{++}).

6) $t \sim 10^{12}$ s at $T \sim T_{RM} \approx 1$ eV. AC-matter dominance starts with (AC)-"atoms", playing the role of CDM in the formation of Large Scale structures.

7) $z \sim 20$. The formation of galaxies starts, triggering (AC) recombination in dense matter bodies.

D. Galaxy formation in the AC-Universe

The development of gravitational instabilities of AC-atomic gas follows the general path of the CDM scenario, but the composite nature of (AC)-atoms leads to some specific difference. In particular, one might expect that particles with a mass $m_{AC} = 200S_2$ GeV should form gravitationally bound objects with the minimal mass

$$M = m_{Pl}(\frac{m_{Pl}}{m_{AC}})^2 \approx 5 \cdot 10^{28}/S_2^2\,\text{g}. \tag{24}$$

However, this estimation is not valid for composite CDM particles, which (AC)-atoms are.

For $S_2 < 6$ the bulk of (AC) bound states appear in the Universe at $T_{fAC} = 0.7S_2$ MeV and the minimal mass of their gravitationally bound systems is given by the total mass of (AC) within the cosmological horizon in this period, which is of the order of

$$M = \frac{T_{RM}}{T_{fAC}}m_{Pl}(\frac{m_{Pl}}{T_{fAC}})^2 \approx 6 \cdot 10^{33}/S_2^3\,\text{g}, \tag{25}$$

where $T_{RM} = 1$ eV corresponds to the beginning of the AC-matter dominated stage.

If these objects, containing $N = 2 \cdot 10^{55} \cdot S_2^{-4}$ (AC)-atoms, separated by the cosmological expansion at $z_s \sim 20$, they have an internal number density

$$n \approx 6 \cdot 10^{-5} \cdot S_2^{-1}(\frac{1 + z_s}{1 + 20})^3\,\text{cm}^{-3}$$

and the size

$$R = (\frac{N}{4\pi n/3})^{1/3} \approx 3 \cdot 10^{19} \cdot S_2^{-1}(\frac{1 + 20}{1 + z_s})\,\text{cm}. \tag{26}$$

At $S_2 > 6$ the bulk of (AC)-atoms is formed only at $T_{OHe} = 60$ keV due to OLe-helium catalysis. Therefore at $S_2 > 6$ the minimal mass is independent of S_2 and is given by

$$M = \frac{T_{RM}}{T_{OHe}}m_{Pl}(\frac{m_{Pl}}{T_{OHe}})^2 \approx 10^{37}\,\text{g}. \tag{27}$$

The size of (AC)-"atoms" is ($Z_A = Z_C = 2$)

$$R_{AC} \sim 1/(Z_A Z_C \alpha k_y m) \sim 1.37 \cdot 10^{-14} \cdot S_2^{-1}k_y^{-1}\,\text{cm}$$

and their mutual collision cross section is about

$$\sigma_{AC} \sim \pi R_{AC}^2 \approx 6 \cdot 10^{-28} \cdot S_2^{-2}k_y^{-2}\,\text{cm}^2. \tag{28}$$

AC-"atoms" can be considered as collision-less gas in clouds with a number density n_{AC} and size R, if $n_{AC}R < 1/\sigma_{AC}$.

At small energy transfer $\Delta E \ll m$ cross section for interaction of AC-atoms with matter is suppressed by the factor $\sim Z^2(\Delta E/m)^2$, being for scattering on nuclei with charge Z and atomic weight A of the order of $\sigma_{ACZ} \sim Z^2/\pi(\Delta E/m)^2\sigma_{AC} \sim Z^2 A^2 10^{-43}\,\text{cm}^2/S_2^2$. Here we take $\Delta E \sim 2Am_p v^2$ and $v/c \sim 10^{-3}$ and find that even for heavy nuclei with $Z \sim 100$ and $A \sim 200$ this cross section does not exceed $4 \cdot 10^{-35}\,\text{cm}^2/S_2^2$. It proves WIMP-like behavior of AC-atoms in the ordinary matter.

The products of incomplete AC binding - OLe-helium and anomalous helium - have much stronger interaction with matter (nuclear and atomic) and need special strategy for their direct experimental search. In particular it should be noted that OLe-helium represents a tiny fraction of dark matter and thus escapes severe constraints [34] on strongly interacting dark matter particles (SIMPs) [34, 35] imposed by the XQC experiment [36].

Mutual collisions of AC-"atoms" determine the evolution timescale for a gravitationally bound system of collisionless AC-gas

$$t_{ev} = \frac{1}{n\sigma_{AC}v} \approx 10^{23} S_2^{17/6} (\frac{1\,\mathrm{cm}^{-3}}{n})^{7/6}\,\mathrm{s}, \tag{29}$$

where the relative velocity $v = \sqrt{GM/R}$ is taken at $S_2 < 6$ for a cloud of mass Eq.(25) and an internal number density n. The timescale Eq.(29) exceeds substantially the age of the Universe even at $S_2 < 6$. Therefore the internal evolution of AC-atomic self-gravitating clouds cannot lead to the formation of dense objects.

E.   Solution for the problem of Anomalous Helium

The main possible danger for the considered cosmological scenario is the over-production of primordial anomalous isotopes. Pre-galactic abundance of anomalous helium (of C-lepton atoms (eeC^{++})) exceeds by up to 10 orders of magnitude the experimental upper limits on its content in terrestrial matter. The only way to solve the problem of anomalous isotopes is to find a possible reason for their low abundance inside the Earth and a solution to this problem implies a mechanism of effective suppression of anomalous helium in dense matter bodies (in particular, inside the Earth). The idea of such suppression, first proposed in [37] and recently realized in [10] is as follows [14].

If anomalous species have an initial abundance relative to baryons ξ_{i0}, their recombination with the rate $\langle \sigma v \rangle$ in a body with baryonic number density n reduces their abundance during the age of the body t_b down to

$$\xi_i = \frac{\xi_{i0}}{1 + \xi_{i0} n \langle \sigma v \rangle t_b}. \tag{30}$$

If $\xi_{i0} \gg 1/(n \langle \sigma v \rangle t_b)$ in the result, the abundance is suppressed down to

$$\xi_i = \frac{1}{n \langle \sigma v \rangle t_b}. \tag{31}$$

To apply this idea to the case of the AC-model, OLe-helium catalysis can be considered as the mechanism of anomalous isotope suppression.

The mechanism of the above mentioned kind can not in principle suppress the abundance of remnants in interstellar gas by more than factor $f_g \sim 10^{-2}$, since at least 1% of this gas has never passed through stars or dense regions, in which such mechanisms are viable. It may lead to the presence of a C^{++} (and A) component in cosmic rays at a level $\sim f_g \xi_i$. Therefore based on the AC-model one can expect the anomalous helium and "antihelium" fractions in cosmic rays

$$\frac{C^{++}}{He} \leq 10^{-10}/f(S_2),$$
$$\frac{A^{--}}{He} \leq 10^{-10}/f(S_2). \tag{32}$$

These predictions are hardly within the reach for future cosmic ray experiments even for $S_2 \sim 1$ and decrease with S_2 as $\propto S_2^{-2}$ for $S_2 < 1.08$ and as $\propto S_2^{-1/2}$ for $S_2 > 1.08$.

The crucial role of the y-attraction comes into the realization of the above mentioned mechanism. The condition of y-charge neutrality makes Ole-helium to follow anomalous helium atoms ($C^{++}e^-e^-$) in their condensation in ordinary matter objects. Due to this condition OLe-helium and anomalous helium can not separate and AC recombination goes on much more effectively, since its rate is given now by (18)

$$\langle \sigma v \rangle \approx 2 \cdot 10^{-12} (\frac{\alpha_y}{1/30})^2 (\frac{300K}{T})^{9/10} S_2^{-11/10} \frac{cm^3}{s}.$$

This increase of recombination rate reduces primeval anomalous helium (and OLe-helium) terrestrial content down to $r \leq 5 \cdot 10^{-30}$.

In the framework of our consideration, interstellar gas contains a significant ($\sim f_g\xi_C$) fraction of (eeC^{++}). When the interstellar gas approaches Solar System, it might be stopped by the Solar wind in the heliopause at a distance $R_h \sim 10^{15}$ cm from the Sun. In the absence of detailed experimental information about the properties of this region we can assume for our estimation following [14] that all the incoming ordinary interstellar gas, containing dominantly ordinary hydrogen, is concentrated in the heliopause and the fraction of this gas, penetrating this region towards the Sun, is pushed back to the heliopause by the Solar wind. In the result, to the present time during the age of the Solar system t_E a semisphere of width $L \sim R_h$ is formed in the distance R_h, being filled by a gas with density $n_{hel} \sim (2\pi R_h^2 v_g t_E n_g)/(2\pi R_h^2 L) \sim 10^8\,\mathrm{cm}^{-3}$. The above estimations show that this region is transparent for (OHe), but opaque for atomic size remnants, in particular, for (eeC^{++}). Owing to the y-interaction both components can thus be stopped in heliopause. Though the Solar wind cannot directly stop heavy (eeC^{++}), the gas shield in the heliopause slows down their income to Earth and suppresses the incoming flux I_C by a factor $S_h \sim 1/(n_{hel}R_h\sigma_{tra})$, where $\sigma_{tra} \approx 10^{-18}S_2\,\mathrm{cm}^2$. So the incoming flux, reaching the Earth, can be estimated as [10, 14]

$$I_C = \frac{\xi_C f_g n_g v_g}{8\pi} S_h \approx \frac{10^{-10}}{f(S_2)}\frac{S_h}{5 \cdot 10^{-5}}(cm^2 \cdot s \cdot ster)^{-1}. \tag{33}$$

Here $n_g \sim 1\,\mathrm{cm}^{-3}$ and $v_g \sim 2 \cdot 10^4\,\mathrm{cm/s}$.

Kinetic equilibrium between interstellar AC-gas pollution and AC recombination in Earth holds [10] their concentration in terrestrial matter at the level

$$n = \sqrt{\frac{j}{\langle\sigma v\rangle}}, \tag{34}$$

where

$$j_A = j_C = j \sim \frac{2\pi I_C}{L} = 2.5 \cdot 10^{-11}S_h/f(S_2)\,\mathrm{cm}^{-3}\,\mathrm{s}^{-1}, \tag{35}$$

within the water-circulating surface layer of thickness $L \approx 4 \cdot 10^5$ cm. Here $I_C \approx 2 \cdot 10^{-6}S_h(\mathrm{cm}^2 \cdot s \cdot ster)^{-1}$ is given by Eq.(33), factor S_h of incoming flux suppression in heliopause can be as small as $S_h \approx 5 \cdot 10^{-5}$ and $\langle\sigma v\rangle$ is given by the Eq.(18). For these values of j and $\langle\sigma v\rangle$ one obtains in water

$$n \le 3.5\sqrt{S_h/f(S_2)}(\frac{1/30}{\alpha_y})S_2^{11/20}\,\mathrm{cm}^{-3}. \tag{36}$$

It corresponds to a terrestrial anomalous helium abundance

$$r \le 3.5 \cdot 10^{-23}\sqrt{S_h/f(S_2)}(\frac{1/30}{\alpha_y})S_2^{11/20},$$

being below the above mentioned experimental upper limits for anomalous helium ($r < 10^{-19}$).

The reduction of the anomalous helium abundance due to AC recombination in dense matter objects is not accompanied by an annihilation, which was the case for U-hadrons in [10], therefore the AC-model escapes the severe constraints [10] on the products of such an annihilation, which follow from the observed gamma background and the data on neutrino and upward muon fluxes.

F. Effects of (OHe) catalyzed processes in the Earth

The first evident consequence of the proposed excess is the inevitable presence of (OHe) in terrestrial matter. (OHe) concentration in the Earth can reach the value (36) for the incoming (OHe) flux, given by Eq.(33). The relatively small size of neutral (OHe) may provide a catalysis of cold nuclear reactions in ordinary matter (much more effectively, than muon catalysis). This effect needs special and thorough nuclear physical treatment. On the other hand, if A^{--} capture by nuclei, heavier than helium, is not effective and does not lead to a copious production of anomalous isotopes, (OHe) diffusion in matter is determined by an elastic collision cross section (38) and may effectively hide OLe-helium from observations.

One can give the following argument for an effective regeneration of OLe-helium in terrestrial matter. OLe-helium can be destroyed in reactions (13). Then free A^{--} are released and owing to a hybrid Auger effect (capture of A^{--} and ejection of ordinary e from the atom with atomic number A and charge of Z of the nucleus) A^{--}-atoms are formed,

in which A^{--} occupies highly an excited level of the $(Z - A^{--})$ system, which is still much deeper than the lowest electronic shell of the considered atom. $(Z - A^{--})$-atomic transitions to lower-lying states cause radiation in the range intermediate between atomic and nuclear transitions. In course of this falling down to the center of the $(Z - A^{--})$ system, the nucleus approaches A^{--}. For $A > 3$ the energy of the lowest state n (given by $E_n = \frac{M\bar{\alpha}^2}{2n^2} = \frac{2Am_pZ^2\alpha^2}{n^2}$) of the $(Z - A^{--})$ system (having reduced mass $M \approx Am_p$) with a Bohr orbit, $r_n = \frac{n}{M\bar{\alpha}} = \frac{n}{2AZm_p\alpha}$, exceeding the size of the nucleus, $r_A \sim A^{1/3}m_\pi^{-1}$, is less, than the binding energy of (OHe). Therefore the regeneration of OLe-helium in a reaction, inverse to (13), takes place. An additional reason for dominantly elastic channel for reactions (13) is that the final state nucleus is created in the excited state and its de-excitation via α-decay can also result in OLe-helium regeneration.

Another effect is the energy release from OLe-helium catalysis of (AC) binding. The consequences of the latter process are not as pronounced as those discussed in [10, 14] for the annihilation of 4th generation hadrons in terrestrial matter, but it might lead to a possible test for the considered model.

In our mechanism the terrestrial abundance of anomalous $(C^{++}ee)$ is suppressed due to the (OHe) catalyzed binding of most of the C from the incoming flux I_C, reaching the Earth. AC binding is accompanied by de-excitation of the initially formed bound (AC) state. To expel ^4He from OLe-helium, this state should have binding energy exceeding $I_{He} = 1.6 MeV$, therefore MeV range γ transitions from the lowest excited levels to the ground state of (AC) with $I_{AC} = 80S_2k_y MeV$ should take place. The danger of gamma radiation from these transitions is determined by the actual magnitude of the incoming flux, which was estimated in subsection III E as Eq.(33).

The stationary regime of (OHe) catalyzed recombination of these incoming C^{++} in the Earth should be accompanied by gamma radiation with the flux $F(E) = N(E)I_Cl_\gamma/R_E$, where $N(E)$ is the energy dependence of the multiplicity of γ quanta with energy E in (AC)-atomic transitions, R_E is the radius of the Earth and l_γ is the mean free path of such γ quanta. At $E > 10 MeV$ one can roughly estimate the flux $F(E > 10MeV) \sim \cdot\frac{10^{-16}}{f(S_2)}\frac{S_h}{5\cdot10^{-5}}(cm^2 \cdot s \cdot ster)^{-1}$, coming from the atmosphere and the surface layer $l_\gamma \sim 10^3 cm$. Even without the heliopause suppression (namely, taking $S_h = 1$) γ radiation from AC binding seems to be hardly detectable.

In the course of (AC) atom formation electromagnetic transitions with $\Delta E > 1$ MeV can be a source of e^+e^- pairs, either directly with probability $\sim 10^{-2}$ or due to development of electromagnetic cascade. If AC recombination goes on homogeneously in Earth within the water-circulating surface layer of the depth $L \sim 4 \cdot 10^5$ cm inside the volume of Super Kamiokande with size $l_K \sim 3 \cdot 10^3$ cm equilibrium AC recombination should result in a flux of e^+e^- pairs $F_e = N_eI_Cl_K/L$, which for $N_e \sim 1$ can be as large as $F_e \sim \cdot\frac{10^{-12}}{f(S_2)}\frac{S_h}{5\cdot10^{-5}}(cm^2 \cdot s \cdot ster)^{-1}$.

Such an internal source of electromagnetic showers in large volume detectors inevitably accompanies the reduction of the anomalous helium abundance due to AC recombination and might give an advantage of experimental tests for the considered model. Their signal might be easily disentangled [14] (above a few MeV range) with respect to common charged current neutrino interactions and single electron tracks, because the tens MeV gamma lead, by pair productions, to twin electron tracks, nearly aligned along their Cerenkov rings. The signal is piling the energy in windows where few atmospheric neutrino and cosmic Super-Novae radiate. The same gamma flux produced is comparable to expected secondaries of tau decay secondaries while showering in air at the horizons edges([38],[39],[40]). The predicted signal strongly depends, however, on the uncertain astrophysical parameters (concentration OLe-helium and anomalous helium in the interstellar gas, their flux coming to Earth etc) as well as on the geophysical details of the actual distribution of OLe-helium and anomalous helium in the terrestrial matter, surrounding large volume detectors.

Direct search for OLe-helium from dark matter halo is not possible in underground detectors due to OLe-helium slowing down in terrestrial matter. Therefore special strategy of such search is needed, which can exploit sensitive dark matter detectors before they are installed under ground. In particular, future superfluid ^3He detector [41] and even its existing few g laboratory prototype can be used to put constraints on the in-falling OLe-helium flux from galactic halo.

G. (OHe) catalyzed formation of (AC)-matter objects inside ordinary matter stars and planets

(AC)-atoms from the halo interact weakly with ordinary matter and can be hardly captured in large amounts by a matter object. However the following mechanism can provide the existence of a significant amount of (AC)-atoms in matter bodies and even the formation of gravitationally bound dense (AC)- bodies inside them.

Inside a dense matter body (OHe) catalyzes C aggregation into (AC)-atoms in the reaction

$$(eeC^{++}) + (A^{--}He) \rightarrow (AC) + He + 2e. \tag{37}$$
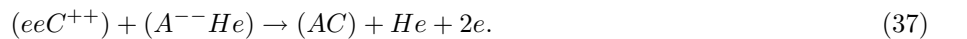
In the result of this reaction (OHe), interacting with matter with a nuclear cross section given by

$$\sigma_{trAb} = \pi R_{OHe}^2 \frac{m_p}{m_A} \approx 10^{-27}/S_2 \, \text{cm}^2, \tag{38}$$

and (eeC^{++}), having a nearly atomic cross section of that interaction

$$\sigma_{tra} = \sigma_a(m_p/m_C) \approx 10^{-18}S_2 \, \text{cm}^2, \tag{39}$$

bind into weakly interacting (AC)-atom, which decouples from the surrounding matter.

In this process "products of incomplete AC-matter combustion" (OLe-helium and anomalous helium), which were coupled to the ordinary matter by hadronic and atomic interactions, convert into (AC) atoms, which immediately sinks down to the center of the body.

The amount of (AC)-atoms produced inside matter object by the above mechanism is determined by the initial concentrations of OLe-helium $(A^{--}He)$ and anomalous helium atoms (eeC^{++}). This amount N defines the number density of (AC)-matter inside the object, being initially $n \sim N/R_s^3$, where R_s is the size of body. At the collision timescale $t \sim (n\sigma_{AC}R_s)^{-1}$, where the (AC)-atom collision cross section is given by Eq.(28), in the central part of body a dense and opaque (AC)-atomic core is formed. This core is surrounded by a cloud of free (AC)-atoms, distributed as $\propto R^{-2}$. Growth and evolution of this (AC)-atomic conglomeration may lead to the formation of a dense self-gravitating (AC)-matter object, which can survive after the star, inside which it was formed, exploded.

The relatively small mass fraction of AC-matter inside matter bodies corresponds to the mass of the (AC)-atomic core $\ll 10^{-4}S_2 M_\odot$ and this mass of AC-matter can be hardly put within its gravitational radius in the result of the (AC)-atomic core evolution. Therefore it is highly improbable that such an evolution could lead to the formation of black holes inside matter bodies.

IV. DISCUSSION

In the present paper we explored the cosmological implications of the AC-model presented in [14, 15] with an additional Coulomb like interaction, mediated by the y-photon. This new $U(1)$ interaction appears naturally in the almost-commutative framework. For the standard model particles the y-photons are invisible, the only source of this invisible light are the AC-particles. Due to this new strict gauge symmetry the AC-leptons acquire stability, similar to the case of 4th generation hadrons [10] and fractons [37].

The AC-particles are lepton like, coupling apart from the y-photons only to the ordinary photon and the Z-boson. Their electric charge is taken to be $-2e$ for the A^{--}-lepton and $+2e$ for the C^{++}-lepton. They may form atom like bound states $(A^{--}C^{++})$ with WIMP like cross section which can play the role of evanescent Cold Dark Matter in the modern Universe. The AC-model escapes most of the drastic problem of the Sinister Universe [12], related with the primordial ^4He cage for -1 charge particles and a consequent overproduction of anomalous hydrogen [13]. These charged ^4He cages pose a serious problems to CDM models with single charged particles, since their Coulomb barrier prevents successful recombination of positively and negatively charged particles. The doubly charged A^{--}-leptons bind with helium in the neutral OLe-helium catalyzers of AC binding and AC-leptons may thus escape this trap.

Nonetheless the binding of AC-leptons into (AC)-"atoms" is a multi step process, which, due to the expansion of the Universe, produces necessarily exotic combinations of AC-matter and ordinary matter, as well as free charged AC-ions. A mechanism to suppress these unwanted remnants is given by the OLe helium catalysis $(AHe) + C \rightarrow (AC) + He$. This process is enhanced by the long-range interaction between the AC-leptons due to the y-photons. It prevents the fractionating of AC-particles and in this way enhances also the binding of AC-particles in dense matter bodies today. This process is necessary to efficiently suppress exotic atoms to avoid the strong observational bounds.

The AC-model with y-interaction may thus solve the serious problem of anomalous atoms, such as anomalous helium, which appeared in the AC-cosmology presented in [14] as well as the question of the stability of the AC-leptons. However the AC-cosmology, even with y-interaction, can only be viewed as an illustration of the possible solution for the Dark Matter problem since the following problems remain open:

1. The reason for particle-antiparticle asymmetry.

The AC-model cannot provide a mechanism to explain the necessary particle-antiparticle asymmetry. Such a mechanism may arise from further extensions of the AC-model within noncommutative geometry or due to phenomena from quantum gravity.

2. Possibly observable nuclear processes due to OLe helium.

A challenging problem is the possible existence of OLe helium (AHe) and of nuclear transformations, catalyzed by (AHe). The question about its consistency with observations remains open, since special nuclear physics analysis is needed to reveal what are the actual OLe-helium effects in SBBN and in terrestrial matter.

3. Recombination of AC-particles in dense matter objects.

The recombination into (AC)-atoms and the consequent release of gamma energy at tens MeV, at the edge of detection in Super Kamiokande underground detector, (at rate comparable to cosmic neutrino Supernovae noise or Solar Flare thresholds [38]). Their signal might be easily disentangled [14] (above a few MeVs) respect common charged current neutrino interactions and single electron tracks because the tens MeV gamma lead, by pair productions, to twin electron tracks, nearly aligned along their Cerenkov rings. The signal is piling the energy in windows where few atmospheric neutrino and cosmic Super-Novae radiate.

4. Mixing of y-photons with neutral gauge bosons.

Due to the interaction of AC-leptons with photons and Z-bosons the invisible y-photons will appear in fermionic AC-loops. Thus standard model fermions may acquire a weak long-range y-interaction. Furthermore it may be necessary to take the AC-lepton loops into account for high precision calculations of QED parameters such as the anomalous magnetic moment of the muon. Since these parameters are extremely well known, they may provide a crucial lower bound for the mass of the AC-particles.

In the context of AC-cosmology search for AC leptons at accelerators acquires the meaning of crucial test for the existence of basic components of the composite dark matter. One can hardly overestimate the significance of positive results of such searches, if AC leptons really exist and possess new long range interaction.

To conclude, in the presence of the y-interaction AC-cosmology can naturally resolve the problem of anomalous helium, avoiding all the observational constraints on the effects, accompanying reduction of its concentration. Therefore the AC-model with invisible light for its dark matter components might provide a realistic model of composite dark matter.

Note added: This paper has been merged with [14] for publication in Class. Quantum Grav. [42].

Appendix 1: Basic definitions of noncommutative geometry

In this section we will give the necessary basic definitions for a classification of almost commutative geometries from a particle physics point of view. As mentioned above only the matrix part will be taken into account, so we restrict ourselves to real, S^0-real, finite spectral triples $(\mathcal{A}, \mathcal{H}, \mathcal{D}, J, \epsilon, \chi)$. The algebra $\mathcal{A}$ is a finite sum of matrix algebras $\mathcal{A} = \oplus_{i=1}^{N} M_{n_i}(\mathbb{K}_i)$ with $\mathbb{K}_i = \mathbb{R}, \mathbb{C}, \mathbb{H}$ where $\mathbb{H}$ denotes the quaternions. A faithful representation ρ of $\mathcal{A}$ is given on the finite dimensional Hilbert space $\mathcal{H}$. The Dirac operator $\mathcal{D}$ is a selfadjoint operator on $\mathcal{H}$ and plays the role of the fermionic mass matrix. J is an antiunitary involution, $J^2 = 1$, and is interpreted as the charge conjugation operator of particle physics. The S^0-real structure ϵ is a unitary involution, $\epsilon^2 = 1$. Its eigenstates with eigenvalue $+1$ are the particle states, eigenvalue -1 indicates antiparticle states. The chirality χ is as well a unitary involution, $\chi^2 = 1$, whose eigenstates with eigenvalue $+1$ (-1) are interpreted as right (left) particle states. These operators are required to fulfill Connes' axioms for spectral triples:

- $[J, \mathcal{D}] = [J, \chi] = [\epsilon, \chi] = [\epsilon, \mathcal{D}] = 0, \quad \epsilon J = -J\epsilon, \quad \mathcal{D}\chi = -\chi\mathcal{D},$
  $[\chi, \rho(a)] = [\epsilon, \rho(a)] = [\rho(a), J\rho(b)J^{-1}] = [[\mathcal{D}, \rho(a)], J\rho(b)J^{-1}] = 0, \forall a, b \in \mathcal{A}.$

- The chirality can be written as a finite sum $\chi = \sum_i \rho(a_i) J\rho(b_i) J^{-1}$. This condition is called *orientability*.

- The intersection form $\cap_{ij} := \operatorname{tr}(\chi \rho(p_i) J\rho(p_j) J^{-1})$ is non-degenerate, $\det \cap \neq 0$. The p_i are minimal rank projections in $\mathcal{A}$. This condition is called *Poincaré duality*.

Now the Hilbert space $\mathcal{H}$ and the representation ρ decompose with respect to the eigenvalues of ϵ and χ into left and right, particle and antiparticle spinors and representations:

$$\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R \oplus \mathcal{H}_L^c \oplus \mathcal{H}_R^c$$

$$\rho = \rho_L \oplus \rho_R \oplus \overline{\rho_L^c} \oplus \overline{\rho_R^c} \tag{40}$$

In this representation the Dirac operator has the form

$$\mathcal{D} = \begin{pmatrix} 0 & \mathcal{M} & 0 & 0 \\ \mathcal{M}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\mathcal{M}} \\ 0 & 0 & \overline{\mathcal{M}^*} & 0 \end{pmatrix}, \tag{41}$$

where $\mathcal{M}$ is the fermionic mass matrix connecting the left and the right handed Fermions.

Since the individual matrix algebras have only one irreducible representation for $\mathbb{K} = \mathbb{R}, \mathbb{H}$ and two for $\mathbb{K} = \mathbb{C}$ (the fundamental one and its complex conjugate), ρ may be written as a direct sum of these fundamental representations with mulitiplicities

$$\rho(\oplus_{i=1}^N a_i) := (\oplus_{i,j=1}^N a_i \otimes 1_{m_{ji}} \otimes 1_{(n_j)}) \ \oplus \ (\oplus_{i,j=1}^N 1_{(n_i)} \otimes 1_{m_{ji}} \otimes \overline{a_j}).$$

There arise certain subtleties which are described in detail in [17, 26, 27] and will be treated in a later extension of our work.

The first summand denotes the particle sector and the second the antiparticle sector. For the dimensions of the unity matrices we have $(n) = n$ for $\mathbb{K} = \mathbb{R}, \mathbb{C}$ and $(n) = 2n$ for $\mathbb{K} = \mathbb{H}$ and the convention $1_0 = 0$. The multiplicities m_{ji} are non-negative integers. Acting with the real structure J on ρ permutes the main summands and complex conjugates them. It is also possible to write the chirality as a direct sum

$$\chi = (\oplus_{i,j=1}^N 1_{(n_i)} \otimes \chi_{ji} 1_{m_{ji}} \otimes 1_{(n_j)}) \ \oplus \ (\oplus_{i,j=1}^N 1_{(n_i)} \otimes \chi_{ji} 1_{m_{ji}} \otimes 1_{(n_j)}),$$

where $\chi_{ji} = \pm 1$ according to our previous convention on left-(right-)handed spinors. One can now define the multiplicity matrix $\mu \in M_N(\mathbb{Z})$ such that $\mu_{ji} := \chi_{ji} m_{ji}$. This matrix is symmetric and decomposes into a particle and an antiparticle matrix, the second being just the particle matrix transposed, $\mu = \mu_P + \mu_A = \mu_P + \mu_P^T$. The intersection form of the Poincaré duality is now simply $\cap = \mu + \mu^T$, see [26, 27].

The mass matrix $\mathcal{M}$ of the Dirac operator connects the left and the right handed Fermions. Using the decomposition of the representation ρ and the corresponding decomposition of the Hilbert space $\mathcal{H}$ we find two types of submatrices in $\mathcal{M}$, namely $M \otimes 1_{(n_k)}$ and $1_{(n_k)} \otimes M$. M is a complex $(n_i) \times (n_j)$ matrix connecting the i-th and the j-th sub-Hilbert space and its eigenvalues give the masses of the fermion multiplet. We will call the k-th algebra the colour algebra.

Appendix 2: Irreducibility, Non-Degeneracy

To classify the almost commutative spectral triples we will impose some extra conditions as in [17]. We will require the spectral triples to be irreducible and non-degenerate according to the following definitions:

Definition IV.1. *i) A spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ is degenerate if the kernel of $\mathcal{D}$ contains a non-trivial subspace of the complex Hilbert space $\mathcal{H}$ invariant under the representation ρ on $\mathcal{H}$ of the real algebra $\mathcal{A}$.*
ii) A non-degenerate spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ is reducible if there is a proper subspace $\mathcal{H}_0 \subset \mathcal{H}$ invariant under the algebra $\rho(\mathcal{A})$ such that $(\mathcal{A}, \mathcal{H}_0, \mathcal{D}|_{\mathcal{H}_0})$ is a non-degenerate spectral triple. If the triple is real, S^0-real and even, we require the subspace $\mathcal{H}_0$ to be also invariant under the real structure J, the S^0-real structure ϵ and under the chirality χ such that the triple $(\mathcal{A}, \mathcal{H}_0, \mathcal{D}|_{\mathcal{H}_0})$ is again real, S^0-real and even.
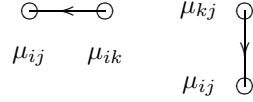
Definition IV.2. *The irreducible spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ is dynamically non-degenerate if all minima $^{\hat{f}}\mathcal{D}$ of the action $V(^f\mathcal{D})$ define a non-degenerate spectral triple $(\mathcal{A}, \mathcal{H}, {}^{\hat{f}}\mathcal{D})$ and if the spectra of all minima have no degeneracies other than the three kinematical degeneracies: left-right, particle-antiparticle and colour. Of course in the massless case there is no left-right degeneracy. We also suppose that the colour degeneracies are protected by the little group. By this we mean that all eigenvectors of $^{\hat{f}}\mathcal{D}$ corresponding to the same eigenvalue are in a common orbit of the little group (and scalar multiplication and charge conjugation).*

In physicists' language non-degeneracy excludes all models with pairwise equal fermion masses in the left handed particle sector up to colour degeneracy. Irreducibility means that we restrict ourselves to one fermion generation and wish to keep the number of fermions as small as allowed by the axioms for spectral triples. The last requirement of definition IV.2 means noncommutative colour groups are unbroken. It ensures that the corresponding mass degeneracies are protected from quantum corrections. It should be noted that the standard model of particle physics meets all these requirements.

Appendix 3: Krajewski Diagrams

Connes' axioms, the decomposition of the Hilbert space, the representation and the Dirac operator allow a diagrammatic dipiction. As was shown in [27] and [17] this can be boiled down to simple arrows, which encode the multiplicity matrix and the fermionic mass matrix. From this information all the ingredients of the spectral triple can be recovered. For our purpose a simple arrow and connections of arrows at one point (i.e. double arrows, edges, etc) are sufficient. The arrows always point from right fermions (positive chirality) to left fermions (negative chirality). We may also restrict ourselves to the particle part, since the information of the antiparticle part is included by transposing the particle part. We will adopt the conventions of [17] so that algebra elements tensorised with $1_{m_{ij}}$ will be written as a direct sum of m_{ij} summands.
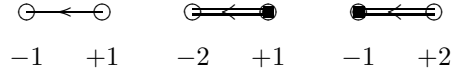
• The Dirac operator: The components of the (internal) Dirac operator are represented by horizontal or vertical lines connecting two nonvanishing entries of opposite signs in the multiplicity matrix μ and we will orient them from plus to minus. Each arrow represents a nonvanishing, complex submatrix in the Dirac operator: For instance μ_{ij} can be linked to μ_{ik} or μ_{kj} by

$$\underset{\mu_{ij}\qquad \mu_{ik}}{\ominus\!\longleftarrow\!\ominus} \qquad\qquad \overset{\mu_{kj}}{\underset{\mu_{ij}}{\ominus\,\big\downarrow\,\ominus}}$$

and these arrows represent respectively submatrices of $\mathcal{M}$ in $\mathcal{D}$ of type $M \otimes 1_{(n_i)}$ with M a complex $(n_j) \times (n_k)$ matrix and $1_{(n_j)} \otimes M$ with M a complex $(n_i) \times (n_k)$ matrix.

The requirement of non-degeneracy of a spectral triple means that every nonvanishing entry in the multiplicity matrix μ is touched by at least one arrow.

• Convention for the diagrams: We will see that (for sums of up to three simple algebras) irreducibility implies that most entries of μ have an absolute value less than or equal to two. So we will use a *simple arrow* to connect plus one to minus one and *double arrows* to connect plus one to minus two or plus two to minus one:)

$$\underset{-1\qquad +1}{\ominus\!\longleftarrow\!\ominus} \qquad\qquad \underset{-2\qquad +1}{\ominus\!\Longleftarrow\!\bullet} \qquad\qquad \underset{-1\qquad +2}{\bullet\!\Longleftarrow\!\ominus}$$

Multiple arrows beginning or ending at one point are with or without edges are built in an obvious way iterating the procedure above. We will give examples below that will clarify these constructions.
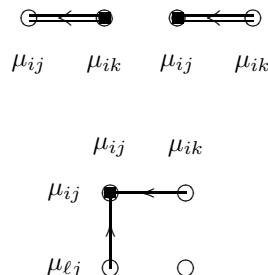
Our arrows always point from plus, that is right chirality, to minus, that is left chirality. For a given algebra, every spectral triple is encoded in its multiplicity matrix which itself is encoded in its Krajewski diagram, a field of arrows. In our conventions, for particles, $\epsilon = 1$, the column label of the multiplicity matrix indicates the representation, the row label indicates the multiplicity. For antiparticles, the row label of the multiplicity matrix indicates the representation, the column label indicates the multiplicity.

Every arrow comes with three algebras: Two algebras that localize its end points, let us call them *right and left algebras* and a third algebra that localizes the arrow, let us call it *colour algebra*. For example for the arrow

$$\underset{\mu_{ij}\qquad \mu_{ik}}{\ominus\!\longleftarrow\!\ominus}$$
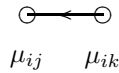
the left algebra is $\mathcal{A}_j$, the right algebra is $\mathcal{A}_k$ and the colour algebra is $\mathcal{A}_i$.

The *circles* in the diagrams only intend to guide the eye. A *black disk* on a multiple arrow indicates that the coefficient of the multiplicity matrix is plus or minus one at this location, "the arrows are joined at this location". For example the the following arrows

$$\underset{\mu_{ij}\qquad \mu_{ik}}{\ominus\!\Longleftarrow\!\bullet} \qquad\qquad \underset{\mu_{ij}\qquad \mu_{ik}}{\bullet\!\Longleftarrow\!\ominus}$$

$$\overset{\mu_{ij}\qquad \mu_{ik}}{\underset{\mu_{\ell j}}{\overset{\mu_{ij}}{\bullet\!\longleftarrow\!\ominus}}\big\uparrow\,\ominus\quad\ominus}$$

represent respectively submatrices of $\mathcal{M}$ of type

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} \otimes 1_{(n_i)} \quad \text{and} \quad \begin{pmatrix} M_1 & M_2 \end{pmatrix} \otimes 1_{(n_i)}$$

with M_1, M_2 of size $(n_j) \times (n_k)$ or in the third case, a matrix of type $\begin{pmatrix} M_1 \otimes 1_{(n_i)} & 1_{(n_j)} \otimes M_2 \end{pmatrix}$ where M_1 and M_2 are of size $(n_j) \times (n_k)$ and $(n_i) \times (n_\ell)$.

According to these rules, we can omit the number $\pm 1, \pm 2$ under the arrows like in figure 2, since they are now redundant.
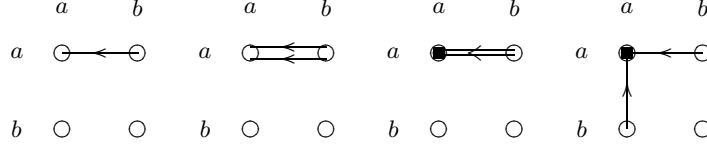

FIG. 2: Four example diagrams

The easiest way to understand the reconstruction of a spectral triple from a diagram is by giving a few examples: Take the algebra $\mathcal{A} = \mathbb{H} \oplus M_3(\mathbb{C}) \ni (a,b)$ with the first diagram of figure 2. Then the multiplicity matrix is

$$\mu = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}.$$

Using (40), its representation is, up to unitary equivalence

$$\rho_L(a,b) = a \otimes 1_2, \ \rho_R(a,b) = b \otimes 1_2, \ \rho_L^c(a,b) = 1_2 \otimes a, \ \rho_R^c(a,b) = 1_3 \otimes a.$$

The Hilbert space is

$$\mathcal{H} = \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^4 \oplus \mathbb{C}^6.$$

In its Dirac operator (41), $\mathcal{M} = M \otimes 1_2$, where M is a nonvanishing complex 2×3 matrix.
Real structure, S^0-real structure and chirality are given by (cc stands for complex conjugation)

$$J = \begin{pmatrix} 0 & 1_{10} \\ 1_{10} & 0 \end{pmatrix} \circ \text{cc}, \quad \epsilon = \begin{pmatrix} 1_{10} & 0 \\ 0 & -1_{10} \end{pmatrix}, \quad \chi = \begin{pmatrix} -1_4 & 0 & 0 & 0 \\ 0 & 1_6 & 0 & 0 \\ 0 & 0 & -1_4 & 0 \\ 0 & 0 & 0 & 1_6 \end{pmatrix}.$$

The first tensor factor in $a \otimes 1_2$ concerns particles, the second concerns antiparticles denoted by $\cdot^c$. The antiparticle representation is read from the transposed multiplicity matrix.

For the second diagram of figure 2 we find the multiplicity matrix

$$\mu = \begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix}.$$

The representation is read off as

$$\rho_L(a,b) = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \otimes 1_2, \quad \rho_R(a,b) = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} \otimes 1_2,$$

$$\rho_L^c(a,b) = \begin{pmatrix} 1_2 & 0 \\ 0 & 1_2 \end{pmatrix} \otimes a, \quad \rho_R^c(a,b) = \begin{pmatrix} 1_3 & 0 \\ 0 & 1_3 \end{pmatrix} \otimes a.$$

The Hilbert space is

$$\mathcal{H} = \mathbb{C}^4 \oplus \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6 \oplus \mathbb{C}^4 \oplus \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6,$$

and for the mass matrix in the Dirac operator we find

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \otimes 1_2,$$

where M_1 and M_2 are nonvanishing complex 2×3 matrices. Real structure, S^0-real structure and chirality are given by

$$J = \begin{pmatrix} 0 & 1_{20} \\ 1_{20} & 0 \end{pmatrix} \circ \text{cc}, \quad \epsilon = \begin{pmatrix} 1_{20} & 0 \\ 0 & -1_{20} \end{pmatrix}, \quad \chi = \begin{pmatrix} -1_8 & 0 & 0 & 0 \\ 0 & 1_{12} & 0 & 0 \\ 0 & 0 & -1_8 & 0 \\ 0 & 0 & 0 & 1_{12} \end{pmatrix}.$$

The third diagram of figure 2 yields

$$\mu = \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix},$$

and its spectral triple reads:

$$\rho_L(a, b) = a \otimes 1_2, \quad \rho_R(a, b) = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} \otimes 1_2,$$

$$\rho_L^c(a, b) = 1_2 \otimes a, \quad \rho_R^c(a, b) = \begin{pmatrix} 1_3 & 0 \\ 0 & 1_3 \end{pmatrix} \otimes a, \tag{42}$$

$$\mathcal{H} = \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6 \oplus \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6, \tag{43}$$

$$\mathcal{M} = \begin{pmatrix} M_1 & M_2 \end{pmatrix} \otimes 1_2, \ M_1 \text{ and } M_2 \text{ of size } 2 \times 3,$$

$$J = \begin{pmatrix} 0 & 1_{16} \\ 1_{16} & 0 \end{pmatrix} \circ \text{cc}, \quad \epsilon = \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix}, \quad \chi = \begin{pmatrix} -1_4 & 0 & 0 & 0 \\ 0 & 1_{12} & 0 & 0 \\ 0 & 0 & -1_4 & 0 \\ 0 & 0 & 0 & 1_{12} \end{pmatrix}.$$

Finally, still for the same algebra, let us consider the last diagram of figure 2. It gives

$$\mu = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix},$$

and

$$\rho_L(a, b) = a \otimes 1_2, \quad \rho_R(a, b) = \begin{pmatrix} b \otimes 1_2 & 0 \\ 0 & a \otimes 1_3 \end{pmatrix},$$

$$\rho_L^c(a, b) = 1_2 \otimes a, \quad \rho_R^c(a, b) = \begin{pmatrix} 1_3 \otimes a & 0 \\ 0 & 1_2 \otimes b \end{pmatrix}, \tag{44}$$

$$\mathcal{H} = \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6 \oplus \mathbb{C}^4 \oplus \mathbb{C}^6 \oplus \mathbb{C}^6, \tag{45}$$

$$\mathcal{M} = \begin{pmatrix} M_1 \otimes 1_2 & 1_2 \otimes M_2 \end{pmatrix}, \quad M_1 \text{ and } M_2 \text{ of size } 2 \times 3.$$

In the four above examples all arrows have left algebra $\mathbb{H}$, right algebra $M_3(\mathbb{C})$ and colour algebra $\mathbb{H}$.

We can exhibit in these simple examples two features that will become central for our algorithm. First, the distinction between a double arrow in the third diagram and an "edge" in the fourth diagram. Although transposing a single arrow amounts just to exchanging particle and anti-particle and is thus of no physical relevance, connecting arrows by building an edge alters the Dirac operator, the Hilbert space and the representation fundamentally. As we can immediately see from the mass matrices the spectral triple with an edge cannot be converted into a spectral triple of two connected parallel arrows. Only the simultaneous transposition of all arrows connected at a point results in a mere exchange of particles and antiparticles. Consequently we have to take "edges" like the fourth diagram into account separately.

Secondly we observe that the first diagram is minimal, if we delete an arrow the determinant of the intersection form will become zero (in fact the whole intersection form will be zero). We can erase an arrow from the other diagrams, dropping back to the first diagram (or its transposed). These diagrams are therefore reducible to the first diagram.

On the other hand the second diagram is reducible to the third diagram, since its spectral triples can be reduced by sizing down the Hilbert space, the left representation and adjusting the mass matrix. The last diagram has a similar relation to the second, but in addition one of its arrows is transposed. It can thus not be found by simply reducing the second diagram, and so we always have to take the edges into account separately. We should perhaps note here that in the case of three and more matrix algebras one arrow will not be sufficient to produce a non-zero determinant of the intersection form.

For the physical content of the diagrams, i.e. for the Yang-Mills-Higgs models they produce, it is irrelevant if we reverse all the arrows at once, which is equivalent to multiplying the multiplicity matrix with -1, or if we permute the algebras. Therefore diagrams that differ only with respect to one of these operations will be considered equivalent. We will always choose only one representative of the set of equivalent diagrams.

Arrows that are superfluous will not be taken into account either. These include two arrows connecting the same algebras and having the same colour algebra but reversed directions. Their contributions to the multiplicity matrix cancel out and thus the spectral triple is reducible. When several arrows are connected, they have to be connected to the same point with common chirality. Otherwise three arrows would get connected in a row and we could erase the middle arrow (i.e. the mass matrix) without altering the multiplicity matrix.

We see that irreducible spectral triples are depicted by minimal (i.e. irreducible) Krajewski diagrams. These have as few arrows as possible which may be further reducible by connecting them or building edges. The aim of the algorithm presented in this paper is to find all irreducible diagrams for a given number of matrix algebras. It should be evident that the number of possibilities to put arrows in a diagram increases factorially with the number of matrix algebras.

Appendix 4: Obtaining the Yang-Mills-Higgs theory

To complete our short survey on the almost-commutative standard model, we will give a brief glimpse on how to construct the actual Yang-Mills-Higgs theory. We started out with the fixed (for convenience flat) Dirac operator of a 4-dimensional spacetime with a fixed fermionic mass matrix. To generate curvature we have to perform a general coordinate transformation and then fluctuate the Dirac operator. This can be achieved by lifting the automorphisms of the algebra to the Hilbert space, unitarily transforming the Dirac operator with the lifted automorphisms and then building linear combinations. Again we restrict ourselves to the finite case. Except for complex conjugation in $M_n(\mathbb{C})$ and permutations of identical summands in the algebra $\mathcal{A} = \mathcal{A}_1 \oplus \mathcal{A}_2 \oplus ... \oplus \mathcal{A}_N$, every algebra automorphism σ is inner, $\sigma(a) = uau^{-1}$ for a unitary $u \in U(\mathcal{A})$. Therefore the connected component of the automorphism group is $\mathrm{Aut}(\mathcal{A})^e = U(\mathcal{A})/(U(\mathcal{A}) \cap \mathrm{Center}(\mathcal{A}))$. Its lift to the Hilbert space [43]

$$L(\sigma) = \rho(u)J\rho(u)J^{-1}$$

is multi-valued.

The *fluctuation* $^f\mathcal{D}$ of the Dirac operator $\mathcal{D}$ is given by a finite collection f of real numbers r_j and algebra automorphisms $\sigma_j \in \mathrm{Aut}(\mathcal{A})^e$ such that

$$^f\mathcal{D} := \sum_j r_j \, L(\sigma_j) \, \mathcal{D} \, L(\sigma_j)^{-1}, \quad r_j \in \mathbb{R}, \ \sigma_j \in \mathrm{Aut}(\mathcal{A})^e.$$

The fluctuated Dirac operator $^f\mathcal{D}$ is often denoted by φ, the 'Higgs scalar', in the physics literature. We consider only fluctuations with real coefficients since $^f\mathcal{D}$ must remain selfadjoint.

To avoid the multi-valuedness in the fluctuations, we allow the entire unitary group viewed as a (maximal) central extension of the automorphism group.

As mentioned in the introduction an almost commutative geometry is the tensor product of a finite noncommutative triple with an infinite, commutative spectral triple. By Connes' reconstruction theorem [44] we know that the latter comes from a Riemannian spin manifold, which we will take to be any 4-dimensional, compact, flat manifold like the flat 4-torus. The spectral action of this almost commutative spectral triple reduced to the finite part is a functional on the vector space of all fluctuated, finite Dirac operators:

$$V(^f\mathcal{D}) = \lambda \, \mathrm{tr}\big[(^f\mathcal{D})^4\big] - \frac{\mu^2}{2} \, \mathrm{tr}\big[(^f\mathcal{D})^2\big],$$

where λ and μ are positive constants [19]. The spectral action is invariant under lifted automorphisms and even under the unitary group $U(\mathcal{A}) \ni u$,

$$V([\rho(u)J\rho(u)J^{-1}]\,{}^f\!\mathcal{D}\,[\rho(u)J\rho(u)J^{-1}]^{-1}) = V({}^f\!\mathcal{D}),$$

and it is bounded from below. To obtain the physical content of a diagram and its associated spectral triple one has to find the minima $\hat{{}^f\!\mathcal{D}}$ of this action and their spectra. But this goes far beyond the scope of this paper and we will content ourselves here with the algorithm to find irreducible Krajewski diagrams.

Appendix 5: Deriving the spectral triple of AC-fermions

The Krajewski diagram of the particle model under consideration encodes an almost-commutative spectral triple with six summands in the internal algebra:
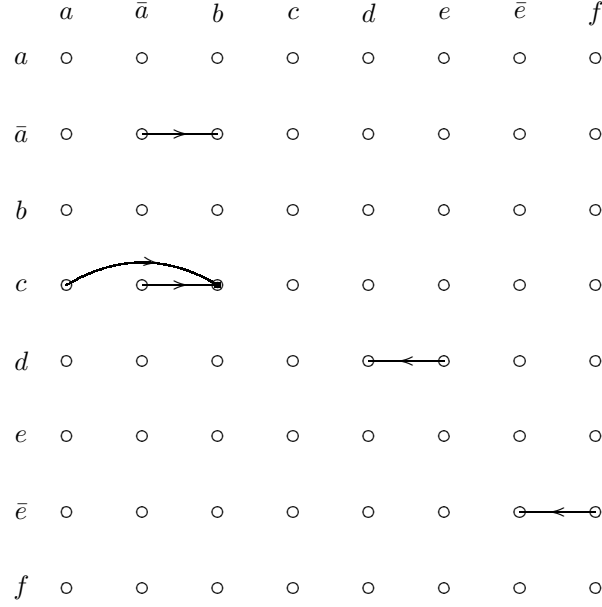
All the necessary translation rules between Krajewski diagrams and the corresponding spectral triples can be found in [27] and [17]. The matrix is already *blown up* in the sense that the representations of the complex parts of the matrix algebra have been fixed. It is the same diagram from which the AC-fermions model of [15] was derived. One can clearly see the sub-diagram of the standard model in the upper 4×4 corner.

To incorporate a new interaction for the AC-fermions the simplest possible extension of the almost-commutative spectral triple [15] will be to extend the quaternion algebra $\mathbb{H}$ to the algebra of complex 2×2-matrices, $M_2(\mathbb{C})$. The notation for the algebra and its elements will be the following:

$$\mathcal{A} = \mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \ni (a, b, c, d, e, f),$$

which has as its representation

$$\rho_L(a,b,c,d,e,f) = \begin{pmatrix} b \otimes 1_3 & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & \bar{e} \end{pmatrix}, \quad \rho_R(a,b,c,d,e,f) = \begin{pmatrix} a1_3 & 0 & 0 & 0 & 0 \\ 0 & \bar{a}1_3 & 0 & 0 & 0 \\ 0 & 0 & \bar{a} & 0 & 0 \\ 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & f \end{pmatrix},$$

$$\rho_L^c(a,b,c,d,e,f) = \begin{pmatrix} 1_2 \otimes c & 0 & 0 & 0 \\ 0 & \bar{a}1_2 & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & e \end{pmatrix}, \quad \rho_R^c(a,b,c,d,e,f) = \begin{pmatrix} c & 0 & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 \\ 0 & 0 & \bar{a} & 0 & 0 \\ 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & \bar{e} \end{pmatrix}.$$

These representations are faithful on the Hilbert space given below and serve as well to construct the lift of the automorphism group. Roughly spoken each diagonal entry of the representation of the algebra can be associated to fermion multiplet. For example the first entry of ρ_L, $b \otimes 1_3$, is the representation of the algebra on the up and down quark doublet, where each quark is again a colour triplet. As pointed out in [31] the commutative sub-algebras of $\mathcal{A}$ which are equivalent to the complex numbers, serve as receptacles for the $U(1)$ subgroups embedded in the automorphism group $U(2) \times U(3)$ of the $M_2((C) \oplus M_3(\mathbb{C})$ matrix algebra. One can easily see that in contrast to the matrix algebra considered in [15] there is now a second $U(1)$ embedded in the automorphism group of the algebra. This new $U(1)$ will be coupled to the AC-fermions only, as a minimal extension of the standard model gauge group. The extended lift is defined by

$$L(v,w) := \rho(\hat{u}, \hat{v}, \hat{w}, \hat{x}, \hat{y}, \hat{z}) J \rho(\hat{u}, \hat{v}, \hat{w}, \hat{x}, \hat{y}, \hat{z}) J^{-1}, \tag{46}$$

with

$$\rho(a,b,c,d,e,f) := \rho_L(a,b,c,d,e,f) \oplus \rho_R(a,b,c,d,e,f) \oplus \rho_L^c(a,b,c,d,e,f) \oplus \rho_R^c(a,b,c,d,e,f),$$

where J in (46) is the real structure of the spectral triple, an anti-unitary operator which coincides with the charge conjugation operator. For the central extension the unitary entries in $L(u,v)$ are defined as

$$
\begin{aligned}
\hat{u} &:= (\det v)^{p_1} (\det w)^{q_1} \\
\hat{v} &:= v(\det v)^{p_2} (\det w)^{q_2} \\
\hat{w} &:= w(\det v)^{p_3} (\det w)^{q_3} \\
\hat{x} &:= (\det v)^{p_4} (\det w)^{q_4} \\
\hat{y} &:= (\det v)^{p_5} (\det w)^{q_5} \\
\hat{z} &:= (\det v)^{p_6} (\det w)^{q_6}
\end{aligned}
$$

and the unitaries $(v,w) \in U(M_2(\mathbb{C}) \oplus M_3(\mathbb{C}))$. The exponents, or central charges, of the determinants will constitute the hypercharges corresponding to the $U(1)$ subgroups of the gauge group. To ensure the absence of harmful anomalies a rather cumbersome calculation, [45, 46], results in the following values for the central charges:

$$
\begin{aligned}
p_1 &\in \mathbb{Q}, \quad q_1 \in \mathbb{Q}, \\
p_2 &= -\frac{1}{2}, \quad q_2 = 0, \\
p_3 &= \frac{p_1}{3}, \quad q_3 = \frac{q_1 - 1}{3}, \\
p_4 &\in \mathbb{Q}, \quad q_4 \in \mathbb{Q}, \\
p_5 &= p_4, \quad q_5 = q_4, \\
p_6 &= -p_4, \quad q_6 = -q_4
\end{aligned}
$$

In the spirit of a minimal extension of the standard model with AC-fermions as presented in [14] the particle content of the model should stay unchanged. Furthermore the standard model fermions should not acquire any new interactions on tree-level. To obtain the standard model hypercharge $U_Y(1)$ one can choose the relevant central charges to be $p_1 := 0$ and $q_1 := -1/2$. Setting $q_4 := -1$ will produce electro-magnetic charges of ∓ 2 for the AC-fermions A^{--} and C^{++}, as required by [14].

Now p_4 can still be chosen freely to be any rational number. If p_4 is taken to be different from zero, the AC-fermions will be furnished with a new interaction generated by the second $U(1)$ sub-group, henceforth called $U_{AC}(1)$, in the group of unitaries of the algebra. Since the AC-fermions do not couple to the Higgs scalar this new gauge group will not be effected by the Higgs mechanism and stays thus unbroken. In the model considered in this paper $p_4 := -1$ for simplicity and the whole gauge group, before and after symmetry breaking is given by

$$U_Y(1) \times SU_w(2) \times SU_c(3) \times U_{AC}(1) \longrightarrow U_{em}(1) \times SU_c(3) \times U_{AC}(1)$$

The Hilbert space is the direct sum of the standard model Hilbert space, for details see [20], and the Hilbert space containing the AC-fermions A^{--} and C^{++}, see [15]:

$$\mathcal{H} = \mathcal{H}_{SM} \oplus \mathcal{H}_{AC},$$

where

$$\mathcal{H}_{AC} \ni \begin{pmatrix} \psi_{AL} \\ \psi_{CL} \end{pmatrix} \oplus \begin{pmatrix} \psi_{AR} \\ \psi_{CR} \end{pmatrix} \oplus \begin{pmatrix} \psi_{AL}^c \\ \psi_{CL}^c \end{pmatrix} \oplus \begin{pmatrix} \psi_{AR}^c \\ \psi_{CR}^c \end{pmatrix}.$$

The wave functions ψ_{AL}, ψ_{CL}, ψ_{AR} and ψ_{CR} are the respective left and right handed Dirac 4-spinors. The initial internal Dirac operator, which is to be fluctuated with the lifted automorphisms is chosen to be the mass matrix

$$
\mathcal{M} = \begin{pmatrix} \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \otimes 1_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_e & 0 & 0 \\ 0 & 0 & 0 & m_A & 0 \\ 0 & 0 & 0 & 0 & m_C \end{pmatrix},
$$

with $m_u, m_d, m_e, m_A, m_C \in \mathbb{C}$.

It should be pointed out that the above choice of the summands of the matrix algebra, the Hilbert space and the Dirac operator is rather unique, if one requires the Hilbert space to be minimal and the fermion masses to be non-degenerate. Fluctuating the Dirac operator and calculating the spectral action gives the usual Einstein-Hilbert action, the Yang-Mills-Higgs action of the standard model and a new part in the Lagrangian for the two AC-fermions as well as a term for the standard gauge potential $\tilde{B}_{\mu\nu}$ of the new $U_{AC}(1)$ sub-group:

$$
\begin{aligned}
\mathcal{L}_{AC} = \; & i\psi_{AL}^* D_A \psi_{AL} + i\psi_{AR}^* D_A \psi_{AR} + m_A \psi_{AL}^* \psi_{AR} + m_A \psi_{AR}^* \psi_{AL} \\
& + i\psi_{CL}^* D_C \psi_{CL} + i\psi_{CR}^* D_C \psi_{CR} + m_C \psi_{CL}^* \psi_{CR} + m_C \psi_{CR}^* \psi_{CL} \\
& - \frac{1}{4} \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu}.
\end{aligned}
$$

The covariant derivative couples the AC-fermions to the $U(1)_Y$ sub-group of the standard model gauge group and to the $U_{AC}(1)$ sub-group,

$$
\begin{aligned}
D_{A/C} = \; & \gamma^\mu \partial_\mu + \frac{i}{2} g' Y_{A/C} \gamma^\mu B_\mu + \frac{i}{2} g_{AC} \tilde{Y}_{A/C} \gamma^\mu \tilde{B}_\mu \\
= \; & \gamma^\mu \partial_\mu + \frac{i}{2} e Y_{A/C} \gamma^\mu A_\mu - \frac{i}{2} g' \sin\theta_w Y_{A/C} \gamma^\mu Z_\mu + \frac{i}{2} g_{AC} \tilde{Y}_{A/C} \gamma^\mu \tilde{B}_\mu,
\end{aligned}
$$

where $\tilde{B}$ is the gauge field corresponding to $U(1)_{AC}$, g_{AC} is the corresponding coupling and $\tilde{Y}_{A/C}$ the almost-commutative hypercharge. From $\tilde{Y}_A = -\tilde{Y}_C = 2p_4$ follows with the choice $p4 = -1$ that $\tilde{Y}_A = -\tilde{Y}_C = -2$. The possible range of the coupling g_{AC} cannot be given by almost-commutative geometry, but has to be fixed by experiment. Furthermore B is the gauge field corresponding to $U(1)_Y$, A and Z are the photon and the Z-boson fields, e is the electro-magnetic coupling and θ_w is the weak angle. The hyper-charge $Y_{A/C} = 2q_4$ of the AC-fermions can be any non-zero fractional number with $Y_A = -Y_C$ so that ψ_A and ψ_C have opposite electrical charge. To reproduce the AC-model $q_4 = -1$ was chosen, as stated above, and so $Y_A = -2$ which results in opposite electro-magnetic charges $\mp 2e$ for the AC-fermions A and C.

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[50] The mechanisms of production of (meta)stable Q (and $\bar{Q}$) hadrons and tera-particles in the early Universe, cosmic rays and accelerators were analyzed in [10, 11, 13] and the possible signatures of their wide variety and existence were revealed.

[51] According to Roman Jackiw [47] Julius Wess told and documented the following: *Like many interesting quantal ideas, the notion that spatial coordinates may not commute can be traced to Heisenberg who, in a letter to Peierls, suggested that a coordinate uncertainty principle may ameliorate the problem of infinite self-energies. ... Evidently, Peierls also described it to Pauli, who told it to Oppenheimer, who told it to Snyder, who wrote the first paper on the subject [48]*

[52] An algebra automorphism is a bijective map from the algebra into itself, which preserves the whole structure of the algebra (i.e. the addition, the multiplication and, if present, the involution). The algebra automorphisms form a group.

[53] These subscripts will be dropped when no confusion will arise from it.